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THE STATUS-TEMPERATURE INDICES OF GRAPHS

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Abstract

Graphical indices are numerical values that describe the entire molecular graph of a chemical compound. They are used in chemical graph theory to measure the physical properties of a chemical compound, its chemical reactivities, and its boiling activities. In this study, we introduce the status-temperature based graphical indices such as sum status-temperature $SST(G)$, product statustemperature $PST(G)$ and difference status-temperature $DST(G)$ index of a graph G. In this paper, many bounds and characterizations on status-temperature graphical indices are obtained and its exact values for some classes graphs are found. Also, we obtain the comparative analysis among the status-temperature based graphical indices of molecular graphs of nucleic acids.

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1. Introduction

By a graph $G = (V, E)$ we mean a simple, connected and without pseudographs (multiedges and loops). The vertex set and edge sets are V and E. The cardinalities are $|V(G)| = n$ and $|E(G)| = m$. The number of edges are incident to the vertex u is called degree of the vertex and is represented by $d_G(u)$. The minimum and maximum number of edges are incident to u called minimum and maximum degree of the vertex and is represented by $\delta = \delta(G)$ and $\Delta = \Delta(G)$. The length of the farthest path between the pair of vertices u and v called distance and is represented by $d_G(u, v)$. The length of the maximum and minimum distance between any two pair of vertices called diameter $diam(G) = d$ and radius $rad(G) = r$ of a graph G, respectively. For any undefined term in this paper, we refer to Harary [6].

In 1959, Frank Harary [5], use the concepts of status in the context of graph theory. In 2014, Aouchiche and Hansen [2] introduce the status of a vertex u in a graph G and is defined as

$$\sigma(u) = \sum_{u \in V(G)} d(u, v). \tag{1.1}$$

For more information on status based graphical index and its application, we refer to [8], [11]-[13], [17, 18], [20]-[23].

In 1988, S. Fajtlowicz [3] initiated the concepts of temperature of a vertex u in a graph G and is defined as

$$T(u) = \frac{d_G(u)}{n - d_G(u)}. \tag{1.2}$$

For more information to temperature based graphical indices and its applications, we refer to [4, 10, 14, 16].

Analogously, we initiate the novel distance (status or transmission) and temperature based graphical indices of a simple connected graph G are defined as follows:

(i) The sum status temperature index of a graph G is defined as

$$SST(G) = \sum_{\{u,v\}\subseteq V(G)} T_\sigma(u) + T_\sigma(v) \tag{1.3}$$

(ii) The product temperature index of a graph G is defined as

$$PST(G) = \sum_{\{u,v\}\subseteq V(G)} T_\sigma(u).T_\sigma(v) \tag{1.4}$$

(iii) The difference status temperature index of a graph G is defined as

$$DST(G) = \sum_{\{u,v\}\subseteq V(G)} |T_\sigma(u) - T_\sigma(v)|, \tag{1.5}$$

where $T_\sigma(u)$ represents the status-temperature of a vertex u and is defined as

$$T_\sigma(u) = \frac{\sigma(u)}{n - \sigma(u)}.$$

In 1947, H. Wiener [25] investigate how pure structural variations impact the boiling point of paraffins. They call it as Wiener index and is defined as

$$W(G) = \sum_{\{u,v\}\subseteq V(G)} d(u,v) = \frac{1}{2} \sum_{u\in V(G)} \sigma(u). \tag{1.6}$$

Lemma 1.1. *[26] Let G be any simple connected graph. Then*

(i) $1 \leq d_G(u) \leq (n-1)$.
(ii) $\delta \leq d_G(u) \leq \Delta$.

Lemma 1.2. *[21, 24] Let G be any connected graph with $diam(G) \leq 2$. Then*

(i) $2(n-1) - d_G(u) \leq \sigma(u) \leq diam(G)(n-1) - (diam(G)-1)d_G(u))$.
(ii) $1 \leq \sigma(u) \leq (n-1)^2$.
(iii) $1 \leq \sigma(u) \leq (n-1)diam(G)$.
(iv) $(n-1) \leq \sigma(u) \leq \dfrac{(n-1)(n+2)}{2} - m$

2. Some specific families of graphs

THEOREM 2.1. *Let G be a r-regular graph with $r \geq 3$. Then*

(i) $SST(G) = \dfrac{n(n-1)(2n-2-r)}{|n-(2n-2-r)|}$.

(ii) $PST(G) = \dfrac{n(n-1)(2n-2-r)^2}{2(n-(2n-2-r))^2}$.

(iii) $DST(G) = 0$.

PROOF. Let G be a r-regular graph with $r \geq 3$. If for each vertex u of a graph G and $\sigma(u) = (2n-2-d_G(u))$, then we have

$$\begin{aligned} SST(G) &= \sum_{\{u,v\}\subseteq V(G)} T_\sigma(u) + T_\sigma(v). \\ &= \sum_{\{u,v\}\subseteq V(G)} \frac{2n-2-d_G(u)}{[n-(2n-2-d_G(u))]} + \frac{2n-2-d_G(v)}{[n-(2n-2-d_G(v))]} \\ &= \sum_{\{u,v\}\subseteq V(G)} \frac{2n-2-r}{[n-(2n-2-r)]} + \frac{2n-2-r}{[n-(2n-2-r)]} \\ &= \frac{n(n-1)(2n-2-r)}{|n-(2n-2-r)|}. \end{aligned}$$

Thus the proof of (i) follows. Similarly, we have the proofs of (ii) and (iii). □

COROLLARY 2.1. *Let G be a complete graph K_n with $n \geq 3$. Then*

(i) $SST(K_n) = n(n-1)^2$.

(ii) $PST(K_n) = \dfrac{n(n-1)^3}{2}$.

(iii) $DST(K_n) = 0$.

COROLLARY 2.2. *Let G be a cycle C_n with $n \geq 3$,*

(i) $SST(C_n) = \begin{cases} \dfrac{n^2(n-1)}{|2-n|}, & \text{if } n \text{ is even} \\ \dfrac{n(n+1)(n-1)^2}{|-n^2+4n+1|}, & \text{if } n \text{ is odd} \end{cases}$

(ii) $PST(C_n) = \begin{cases} \dfrac{n^3(n-1)}{2(n^2-n+4)}, & \text{if } n \text{ is even} \\ \dfrac{n(n-1)(n^2-1)^2}{2(n^4-8n^3+14n^2+8n+1))}, & \text{if } n \text{ is odd} \end{cases}$

(iii) $DST(C_n) = 0$.

3. Bounds interms of order, size and minimum/maximum degree

THEOREM 3.1. *Let G be a connected graph. Then*

(i) $\dfrac{n(n-1)(2n-3)}{|3-n|} \leq SST(G) \leq n(n-1)^2$.

(ii) $\dfrac{n(n-1)(2n-3)^2}{2(3-n)^2} \leq PST(G) \leq \dfrac{n(n-1)^3}{2}$.
(iii) $DST(G) = 0$.

PROOF. By the definition of $SST(G)$ and Lemma 1.1 (i), we have

$$SST(G) = \sum_{\{u,v\}\subseteq V(G)} \frac{2n-2-d_G(u)}{[n-(2n-2-d_G(u))]} + \frac{2n-2-d_G(v)}{[n-(2n-2-d_G(v))]}$$
$$\leq \sum_{\{u,v\}\subseteq V(G)} \frac{2n-2-1}{[n-(2n-2-1)]} + \frac{2n-2-1}{[n-(2n-2-1)]}.$$
$$\leq \frac{n(n-1)(2n-3)}{|3-n|}, \quad and$$

$$SST(G) \leq \sum_{\{u,v\}\subseteq V(G)} \frac{2n-2-(n-1)}{[n-(2n-2-(n-1))]} + \frac{2n-2-(n-1)}{[n-(2n-2-(n-1))]}.$$
$$\leq n(n-1)^2.$$

Thus the proof of (i) follows. Similarly, we have the proofs of (ii) and (iii). □

THEOREM 3.2. *Let G be a connected graph. Then*

(i) $n \leq SST(G) \leq \dfrac{n(n-1)^3}{|n-(n-1)^2|}$.
(ii) $\dfrac{n}{2(n-1)} \leq PST(G) \leq \dfrac{n(n-1)^5}{2(n-(n-1)^2)^2}$.
(iii) $DST(G) = 0$.

PROOF. By definition of $SST(G)$ and Lemma 1.2 (ii), we have

$$SST(G) = \sum_{\{u,v\}\subseteq V(G)} \frac{\sigma(u)}{n-\sigma(u)} + \frac{\sigma(v)}{n-\sigma(v)}$$
$$\geq \sum_{\{u,v\}\subseteq V(G)} \frac{1}{n-1} + \frac{1}{n-1} = n, \quad and$$

$$SST(G) \leq \sum_{\{u,v\}\subseteq V(G)} \frac{(n-1)^2}{n-(n-1)^2} + \frac{(n-1)^2}{n-(n-1)^2}.$$
$$\leq \frac{n(n-1)^3}{|n-(n-1)^2|}.$$

Thus the proof of (i) follows. Similarly, we have the prrofs of (ii) and (iii). □

THEOREM 3.3. *Let G be a connected graph. Then*

(i) $n(n-1)^2 \leq SST(G) \leq \dfrac{n(n-1)[(n-1)(n+2)-2m]}{2(m+n)-[(n-1)(n+2)]}$.

(ii) $\dfrac{n(n-1)^3}{2} \leq PST(G) \leq \dfrac{n(n-1)}{2}\Big[\dfrac{n^2+n-2m-2}{2m-n(n-1)+2}\Big]^2.$

(iii) $DST(G) = 0.$

Proof. By the definition of $SST(G)$ and Lemma 1.2 (iv), we have

$$SST(G) = \sum_{\{u,v\}\subseteq V(G)} \frac{\sigma(u)}{n-\sigma(u)} + \frac{\sigma(v)}{n-\sigma(v)}$$

$$\leq \sum_{\{u,v\}\subseteq V(G)} \left[\frac{\frac{(n-1)(n+2)}{2}-m}{n-\left(\frac{(n-1)(n+2)}{2}-m\right)}\right] + \left[\frac{\frac{(n-1)(n+2)}{2}-m}{n-\left(\frac{(n-1)(n+2)}{2}-m\right)}\right]$$

$$\leq \frac{n(n-1)[(n-1)(n+2)-2m]}{2(m+n)-[(n-1)(n+2)]}, \textit{ and}$$

$$SST(G) \geq \sum_{\{u,v\}\subseteq V(G)} \frac{(n-1)}{n-(n-1)} + \frac{(n-1)}{n-(n-1)} = n(n-1)^2.$$

Thus the proof of (i) follows. Similarly, we have the proofs of (ii) and (iii). □

Theorem 3.4. *Let G be a connected graph. Then*

(i) $\dfrac{n(n-1)(2(n-1)-\delta)}{|2-n+\delta|} \leq SST(G) \leq \dfrac{n(n-1)(2(n-1)-\Delta)}{|2-n+\Delta|}.$

(ii) $\dfrac{n(n-1)(2(n-1)-\delta)^2}{2|2-n+\delta|^2} \leq PST(G) \leq \dfrac{n(n-1)(2(n-1)-\Delta)^2}{2|2-n+\Delta|^2}.$

(iii) $DST(G) = 0.$

Proof. By the definition of $SST(G)$ and Lemma 1.1 (ii), we have

$$SST(G) = \sum_{\{u,v\}\subseteq V(G)} \frac{2n-2-d_G(u)}{[n-(2n-2-d_G(u))]} + \frac{2n-2-d_G(v)}{[n-(2n-2-d_G(v))]}$$

$$\geq \sum_{\{u,v\}\subseteq V(G)} \frac{2n-2-\delta}{[n-(2n-2-\delta)]} + \frac{2n-2-\delta}{[n-(2n-2-\delta)]}$$

$$\geq \frac{n(n-1)(2n-2-\delta)}{|2-n+\delta|}, \textit{ and}$$

$$SST(G) \leq \sum_{\{u,v\}\subseteq V(G)} \frac{2n-2-\Delta}{[n-(2n-2-\Delta)]} + \frac{2n-2-\Delta}{[n-(2n-2-\Delta)]}$$

$$SST(G) \leq \frac{n(n-1)(2(n-1)-\Delta)}{|2-n+\Delta|}.$$

Thus the proof of (i) follows. Similarly, we have the proofs of (ii) and (iii). □

THEOREM 3.5. *Let G be a connected graph with $diam(G) \leq 2$. Then*

(i)

$$\frac{n(n-1)(2n-2-\Delta)}{|-n+2+\Delta|} \leq SST(G) \leq \frac{n(n-1)[diam(G)(n-1)-(diam(G)-1)\delta]}{|n-[diam(G)(n-1)-(diam(G)-1)\delta]|}.$$

(ii)

$$\frac{n(n-1)(2n-2-\Delta)^2}{2|-n+2+\Delta|^2} \leq PST(G) \leq \frac{n(n-1)[diam(G)(n-1)-(diam(G)-1)\delta]^2}{2|n-[diam(G)(n-1)-(diam(G)-1)\delta]|^2}.$$

(iii) $DST(G) = 0$.

PROOF. By the definition of $SST(G)$ and Lemma 1.2 (i) with $diam(G) \leq 2$, we have

$$\begin{aligned} SST(G) &\geq \sum_{\{u,v\}\subseteq V(G)} \frac{2n-2-d_G(u)}{|n-(2n-2-d_G(u))|} + \frac{2n-2-d_G(v)}{|n-(2n-2-d_G(v))|} \\ &\geq \sum_{\{u,v\}\subseteq V(G)} \frac{2n-2-\Delta}{|n-(2n-2-\Delta)|} + \frac{2n-2-\Delta}{|n-(2n-2-\Delta)|} \\ &\geq \frac{n(n-1)(2n-2-\Delta)}{|-n+2+\Delta|}, \text{ and} \end{aligned}$$

$$\begin{aligned} SST(G) &\leq \sum_{\{u,v\}\subseteq V(G)} \frac{diam(G)(n-1)-(diam(G)-1)d_G(u)}{|n-(diam(G))(n-1)-(diam(G)-1)d_G(u)|} \\ &+ \frac{diam(G)(n-1)-(diam(G)-1)d_G(u)}{|n-(diam(G))(n-1)-(diam(G)-1)d_G(u)|} \\ &\leq \frac{n(n-1)[diam(G)(n-1)-(diam(G)-1)\delta]}{|n-[diam(G)(n-1)-(diam(G)-1)\delta]|}. \end{aligned}$$

Thus the proof of (i) follows. Similarly, we have the proofs of (ii) and (iii). □

THEOREM 3.6. *Let G be a connected graph with $diam(G) \leq 2$. Then*

(i) $n \leq SST(G) \leq \dfrac{n(n-1)^2 diam(G)}{n(n-1)diam(G)}$.

(ii) $\dfrac{n}{2(n-1)} \leq PST(G) \leq \dfrac{n(n-1)^3 diam(G)^2}{2|n-(n-1)diam(G)|^2}$.

(iii) $DST(G) = 0$.

PROOF. By the definition of $SST(G)$, $PST(G)$ $DST(G)$ and Lemma 1.2 (iii) with $diam(G) \leq 2$, we have the desired results. □

Corollary 3.1. *Let G be a connected graph. Then*

(i) $n \leq SST(G) \leq \dfrac{n(n-1)rad(G)}{|n-(n-1)rad(G)|}$.

(ii) $\dfrac{n}{2(n-1)} \leq PST(G) \leq \dfrac{n(n-1)^3 rad(G)^2}{2|n-(n-1)rad(G)|^2}$.

(iii) $DST(G) = 0$.

4. Bounds among the status-temperature indices

Theorem 4.1. *Let G be a connected graph. Then*

$$\frac{2[n-(n-1)^2]}{(n-1)^2}PST(G) \leq SST(G) \leq 2(n-1)PST(G).$$

Proof. Let G be a connected graph. Then

$$T_\sigma(u) + T_\sigma(v) = T_\sigma(u).T_\sigma(v)\Big[\frac{1}{T_\sigma(u)} + \frac{1}{T_\sigma(v)}\Big]$$

By using the Lemma1.2 (ii), we have

$$\frac{2[n-(n-1)^2]}{(n-1)^2}T_\sigma(u).T_\sigma(v) \leq T_\sigma(u) + T_\sigma(v) \leq 2(n-1)T_\sigma(u).T_\sigma(v).$$

Taking summation to all over the inequalities and for each $\{u,v\} \subseteq V(G)$, we have

$$\frac{2[n-(n-1)^2]}{(n-1)^2}\sum_{\{u,v\}\subseteq V(G)} T_\sigma(u).T_\sigma(v) \leq \sum_{\{u,v\}\subseteq V(G)} T_\sigma(u) + T_\sigma(v)$$
$$\leq 2(n-1)\sum_{\{u,v\}\subseteq V(G)} T_\sigma(u).T_\sigma(v).$$

Therefore,

$$\frac{2[n-(n-1)^2]}{(n-1)^2}PST(G) \leq SST(G) \leq 2(n-1)PST(G).$$

Hence, the proof. □

Theorem 4.2. *Let G be a connected graph with $diam(G) \leq 2$. Then*

$$\frac{2[n-(n-1)diam(G)]}{(n-1)diam(G)}PST(G) \leq SST(G) \leq 2(n-1)PST(G).$$

Proof. Let G be a connected graph. Then

$$T_\sigma(u) + T_\sigma(v) = T_\sigma(u).T_\sigma(v)\Big[\frac{1}{T_\sigma(u)} + \frac{1}{T_\sigma(v)}\Big]$$

By using the Lemma1.2 (iii), we have

$$\frac{2[n-(n-1)diam(G)]}{(n-1)diam(G)}T_\sigma(u).T_\sigma(v) \leq T_\sigma(u) + T_\sigma(v) \leq 2(n-1)T_\sigma(u).T_\sigma(v).$$

Taking summation to all over the inequalities and for each $\{u, v\} \subseteq V(G)$, we have

$$\frac{2[n-(n-1)diam(G)]}{(n-1)diam(G)} \sum_{\{u,v\}\subseteq V(G)} T_\sigma(u).T_\sigma(v) \leq \sum_{\{u,v\}\subseteq V(G)} T_\sigma(u) + T_\sigma(v)$$
$$\leq 2(n-1) \sum_{\{u,v\}\subseteq V(G)} T_\sigma(u).T_\sigma(v).$$

Hence, the proof. □

COROLLARY 4.1. *Let G be a connected graph. Then*

$$\frac{2[n-(n-1)rad(G)]}{(n-1)rad(G)} PST(G) \leq SST(G) \leq 2(n-1)PST(G).$$

THEOREM 4.3. *Let G be a connected graph. Then*

$$DST(G) \leq PST(G)\Big[(n-1) - \frac{|-n^2+3n-1|}{(n-1)^2}\Big].$$

PROOF. Let G be a connected graph. Then

$$|T_\sigma(u) - T_\sigma(v)| \leq T_\sigma(u).T_\sigma(v)\Big[\frac{1}{T_\sigma(u)} - \frac{1}{T_\sigma(v)}\Big]$$
$$\leq T_\sigma(u).T_\sigma(v)\Big[\frac{n-\sigma(v)}{\sigma(v)} - \frac{n-\sigma(u)}{\sigma(u)}\Big]$$

Taking summation to all over the inequalities and for each $\{u, v\} \subseteq V(G)$. By Lemma1.2 (ii). We have

$$DST(G) \leq PST(G)\Big[(n-1) - \frac{|-n^2+3n-1|}{(n-1)^2}\Big].$$

Hence, the proof. □

To prove our next results, we make use of the following definitions of the first and second status connectivity indices [19] are defined by

$S_1(G) = \sum_{\{u,v\}\subseteq V(G)} (\sigma(u)+\sigma(v))$ and $S_2(G) = \sum_{\{u,v\}\subseteq V(G)} \sigma(u)\, \sigma(v)$. Also, the Albertson index or irregular index [1] of a graph G is defined by

$$irr(G) = \sum_{\{u,v\}\subseteq V(G)} |d_G(u) - d_G(v)|.$$

THEOREM 4.4. *Let G be a connected graph. Then*

(i) $SST(G) \leq \dfrac{2nS_1(G) - 4S_2(G)}{n(n-1)[n^2 - 2nS_1(G) + 2S_2(G)]}$.

(ii) $PST(G) \leq \dfrac{2S_2(G)}{|n^3(n-1) - 2nS_1(G) + 2S_2(G)|}$.

(iii) $DST(G) \leq \dfrac{n^2(n-1)irr(G)}{2[n^2 - nS_1(G) + S_2(G)]}$.

Proof. Let G be a connected graph. Then

(i) By the definition of $SST(G)$, we have

$$SST(G) = \sum_{\{u,v\}\subseteq V(G)} \Big[\frac{\sigma(u)}{|n-\sigma(v)|} + \frac{\sigma(v)}{|n-\sigma(v)|}\Big]$$
$$= \sum_{\{u,v\}\subseteq V(G)} \Big[\frac{n[\sigma(u)+\sigma(v)] - 2\sigma(u).\sigma(v)}{|n^2 - n[\sigma(u)+\sigma(v)] + \sigma(u).\sigma(v)|}\Big]$$
$$\leq \frac{2nS_1(G) - 4S_2(G)}{|n^3(n-1) - 2nS_1(G) + 2S_2(G)|}.$$

(ii) By the definition of $PST(G)$, we have

$$PST(G) = \sum_{\{u,v\}\subseteq V(G)} \Big[\frac{\sigma(u)}{|n-\sigma(v)|}\Big].\Big[\frac{\sigma(v)}{|n-\sigma(v)|}\Big].$$
$$\leq \frac{2S_2(G)}{|n^3(n-1) - 2nS_1(G) + 2S_2(G)|}.$$

(iii) By the definition of $DST(G)$, we have

$$DST(G) = \sum_{\{u,v\}\subseteq V(G)} \Big[\frac{\sigma(u)}{|n-\sigma(v)|} - \frac{\sigma(v)}{|n-\sigma(v)|}\Big].$$

$$= \sum_{\{u,v\}\subseteq V(G)} \frac{n[\sigma(u)-\sigma(v)]}{|n^2 - n[\sigma(u)+\sigma(v)] + \sigma(u).\sigma(v)|} \leq \frac{n^2(n-1)irr(G)}{2[n^2 - nS_1(G) + S_2(G)]}.$$

Hence, the proof. □

5. chemical applicabilities for molecular graphs of nucleic acids

Biochemistry is a branch of science that focuses on the study of the chemical processes and substances that occur within living organisms (Human beings). It investigates the structure, function, and metabolism of biomolecules such as carbohydrates, lipids, proteins, enzymes, nucleic acids and their metabolic pathways. Biochemistry also explores genetics, gene expression, and signal transduction, providing insights into cellular communication and regulation.

Here, we can use nucleic acids such as thymine-G_1, asparagine-G_2, glutamine-G_3, and histidine-G_4 are as shown in Figure 1. These nucleic acids have the same exact mass as species in the hydrolyzed tholin sample, with relatively strong intensities in the negative ion mood. For more information we refer [7, 9, 15].

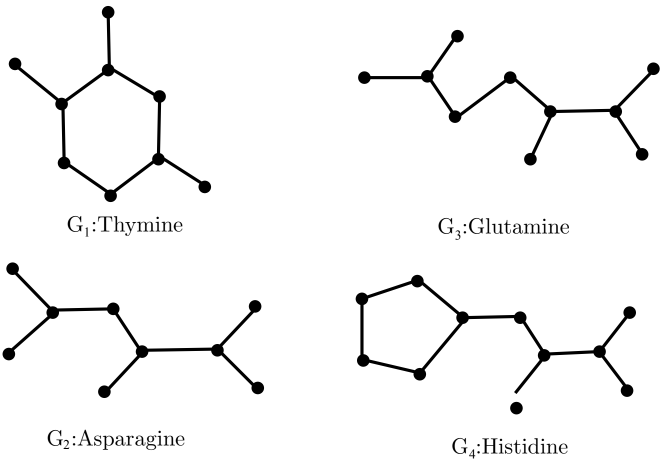

FIGURE 1. Molecular graph of Nucleic acids.

Molecular graph G_i	Status temperature vertex of u										
	$T_\sigma(v_1)$	$T_\sigma(v_2)$	$T_\sigma(v_3)$	$T_\sigma(v_4)$	$T_\sigma(v_5)$	$T_\sigma(v_6)$	$T_\sigma(v_7)$	$T_\sigma(v_8)$	$T_\sigma(v_9)$	$T_\sigma(v_{10})$	$T_\sigma(v_{11})$
G_1	1.64	2.28	2.12	2	2.12	1.6	2.12	2.8	1.69	-	-
G_2	1.52	1.9	1.52	2.28	2.5	1.69	2	1.56	1.56	-	-
G_3	1.42	1.62	1.42	1.83	2	2	1.55	1.71	1.45	1.45	-
G_4	1.61	1.45	1.45	1.61	1.92	2	1.92	1.52	1.64	1.42	1.42

TABLE 1. The value of status-temperature of each vertices

Molecular graph	**Status temperature indices**		
	$SST(G)$	$PST(G)$	$DST(G)$
G_1	39.01	41.99	3.87
G_2	31.61	31.273	3.55
G_3	30.94	26.70	2.04
G_4	37.35	31.80	2.53

TABLE 2. The computed values of status-temperature indices.

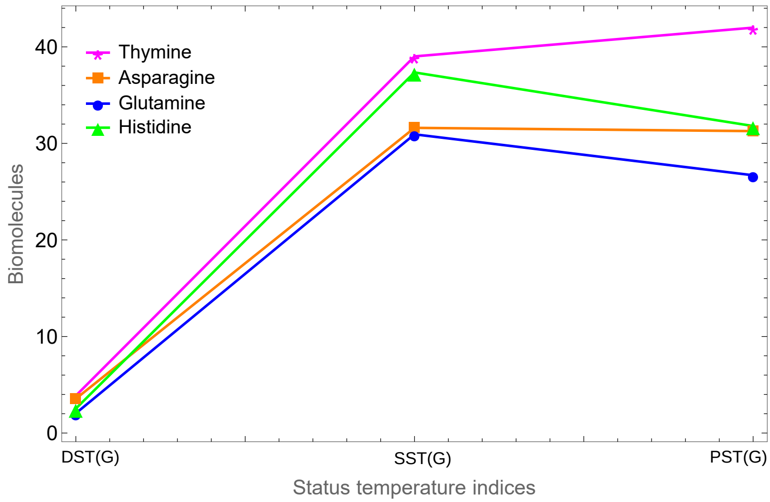

FIGURE 2. Comparative analysis of status-temperature indices of molecular graphs of nucleic acids.

Comparative Analysis: The molecular graphs of nucleic acids contains the number of bonds (edges) and atoms (vertices). Then the values of status-temperature of each atoms of nucleic acids are as shown in the Table 1. By substituting all these values to the eccentric related graphical indices, we obtain Table 2. From this table we obtain the graphical comparison of molecular graphs of nucleic acids as shown in Figure 2. On comparison among the status-temperature graphical indices of molecular graphs of nucleic acids (see, Figure 1), the thymine molecules (i.e., the molecular graph G_1) having more values and glutamine molecules (i.e., the molecular graph G_3) having less value. Therefore, in general, we concluded that the values among the status-temperature graphical indices are slightly oscillating between them.

6. Conclusion and further scope

In this paper, we determined the exact values for some specific classes of graphs and found some bounds in terms of order, size, and degrees. Also, we obtained bounds of the status-temperature based graphical indices. We shows the relationship between the status-temperature graphical indices and molecular graphs of nucleic acids. Further, the comparative advantages, applications and mathematical point of view, many questions are suggested by this research, among them are following.

1. Find the extremal values and extremal graphs of the status-temperature based graphical indices.
2. Characterize among the graphical indices of $SST(G)$, $PST(G)$ and $DST(G)$. Also, explore some results towards QSPR / QSAR / QSTR models.

7. Declaration

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Conflicts of Interest: The authors declares that there are no conflicts of interest.

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