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GEOMETRICAL TRANSFORMATIONS FROM BAUDHĀYANA ŚULBA SŪTRA AND FORMATION OF CITIS

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Abstract

Ancient Indians exhibited proficiency in performing essential numerical computations and practical geometric constructions for the urban civilizations of the Indus Valley. This paper explores various geometric transformations and their utilization in shaping cities, drawing insights from the Baudhāyana Śulba Sūtra, one of the earliest mathematical texts from ancient Indian treatises. Additionally, we assess the precision of these transformations.

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1. Introduction

The mathematical contributions of ancient Hindus can be categorized into three distinct periods: the Vedic age, the pre-medieval period, and the medieval period. The earliest extant work is the Veda, consisting of hymns venerating the supreme. As per M. Winternitz's "The History of Indian Literature" (p.p 310), the era of Veda and Vedāṅga is typically dated from 1500 BCE to 750 BCE. The progress during this period is documented in the Vedāṅga, which serves as an auxiliary to the Veda. According to available treatises, there are six Vedāṅga, as follows:

(i) शक्षा (Śikṣā) – the field of study concerned with accurate articulation and pronunciation is referred to as Phonetics
(ii) छंदस् (Chamdas) – the science that deals with Prosody
(iii) व्याकरण (Vyākaraṇa) – the set of rules governing the formation of words and the combination of words to create sentences i.e., Grammar
(iv) निरुक्त (Nirukta) – the study of Etymology

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(v) ज्योतिषि (Jyotiş) – the study of the motion of Nine planets of the solar system i.e., Astronomy
(vi) कल्प (Kalpa) – the set of rules required to perform the rituals and the ceremonials.

The two Vedāńga mentioned last in the list, namely ज्योतिषि (Jyotiş) and कल्प (Kalpa), encompass the entirety of mathematical development, with a primary focus on geometrical concepts, during that period. Jyotiş played a crucial role for ancient Indians in determining auspicious days and times for conducting rituals, ceremonies, or sacrifices to attain desired outcomes. Two recensions of Jyotiş are identified in ancient treatises - Ārca Jyotiş, associated with the ṚgVeda, and Yājur Jyotiş, linked to the Yājur Veda. These recensions provided guidance on celestial matters to facilitate the optimal timing of various spiritual and ceremonial activities. According to Vedāńga Jyotiş, mathematics has held paramount significance among all branches of sciences since the Vedic period. This emphasis is vividly expressed in the following shloka:

यथा शखा मयूराणां , नागानां मणयो यथा।
तद्वेदांग शास्त्राणां , गणतिं मूर्धनि वर्तते।।

"Like the crests on the heads of peacocks, like the gems on the hoods of cobras, mathematics stands at the pinnacle of the Vedic shastra." The Kalpa Sūtra delves into the regulations and procedures governing Vedic rituals, ceremonies, and sacrifices. It is intricately divided into three sections, namely Śrauta Sūtra, Gṛhya Sūtra, and Dharma. These sections provide detailed instructions for various aspects of Vedic practices, encompassing the rituals performed in public ceremonies (Śrauta Sūtras), domestic rituals and responsibilities (Gṛhya Sūtra), and broader ethical and legal principles (Dharma). Śulba Sūtra are the reference manuals that are used to plan, measure, align, layout, compute, and construct Vedic Yajna Vedi (citi) which are structurally stable and required for spiritually superior performance of the Yajna. Śulba Sūtra is a part of the Śrauta Sūtra section of the Kalpa, which belongs to one of the six Vedāńga (given above) attached to the holy Vedas. The Śulba Sūtra provides spatial and directional parameters that are required for the construction of the Yajna Vedi (citi) while the time-keeping calculations in the Vedic astrology, decide the temporal parameters of the sacrifice. The position vectors and coordinates consisting of orientation, location, and time (diķ - dişa - kāla) can be decided using these texts. "Śulba" primarily means measurement, which is done using the measuring cord called the Rajju or Rope, although sometimes these two terms are used as synonyms. An early Hindu term is also mentioned in the Śulba Sūtra which is Śulba Vijñaña (or Śulba Vigyana), roughly the 'Science of the Śulba' or measurement science, or the geometrical sciences, which also became known later as kşetra ganita. The Vedic geometry originated and developed in a very remote era, which was related to the creations and constructions of the

Yajna Vedi for Vedic yajna. This science is mainly of India and it was developed for the purpose of which no parallel example is found in human history of any other country. There are Seven Śulba Sūtra available today, among them, Baudhāyana Śulba Sūtra is the earliest.

2. Introduction to Fire – Altars

As mentioned in ancient vedic treatises, there are two types of yagna rituals; Nityāgni (नत्यिागन्ा) and Kāmyāgni (काम्यागन्ा). 'Agni' originally means fire but later on it becomes the synonym for fire – altar (vedi). Nityāgni have been done at daily basis while Kāmyāgni were done for the purpose of fulfilling some special expectations. The Kāmyāgni was related to different fire altars of different shapes (such as the shape of a hawk bird with a square body or with bent wings or curved wings, an isosceles triangle, a rhombus, and a chariot wheel), but having an equal area of $7\frac{1}{2}$ sq. puruṣa. These differently shaped fire – altars are known as citi (चति).

3. Beauty of Citi

The word citi is similar to the word cit (चत्त) which refers to consciousness. Each citi comprises a low platform constructed with meticulous precision, consisting of five layers of carefully shaped and arranged bricks. The designs vary, ranging from simple shapes like squares, circles, or rhombuses to more intricate patterns such as a flying falcon with gracefully curved wings, a chariot wheel with or without spokes, or a tortoise with an extended head and legs. These elaborate designs not only showcase artistic beauty but also convey powerful and symbolic meanings. For instance, the falcon symbolizes a majestic bird capable of soaring to heaven, the wheel represents the 'wheel of life' or maybe 'dharma chakra,' and the tortoise serves as a symbol of stability and perseverance.

4. Some Geometrical Transformations

In the Śulba Sūtra, many geometric constructions commence with the establishment of a line known as 'prāci' extending in the east-west direction. This line later plays a crucial role in the final geometric figure, often serving as the center line or line of symmetry. Now, let's explore some of the key transformations outlined in the Baudhāyana Śulba Sūtra.

4.1. Transformation of a square into a rectangle of equal area

समचतुरश्रं दीर्घचतुरश्रंचकीिर्षंस्तदक्ष्णापच्छिद्य भागं द्वेषा वभिज्य पाश्र्वयोरूपदध्याद्यथायोगम् ।। (BSS. 1-52)

To convert a square ABCD into a rectangle Baudhāyana suggested dividing the square into two parts by drawing one of its diagonals AC. Now, further divide one part of the square into two parts. Then, we get two isosceles triangles (ΔEBC and ΔEBA). Now, place these two isosceles triangles on the adjacent edges of the first part of the square i.e., one on the northern side and the other on the eastern side, then we get the desired rectangle ACFG (as shown in figure 1).

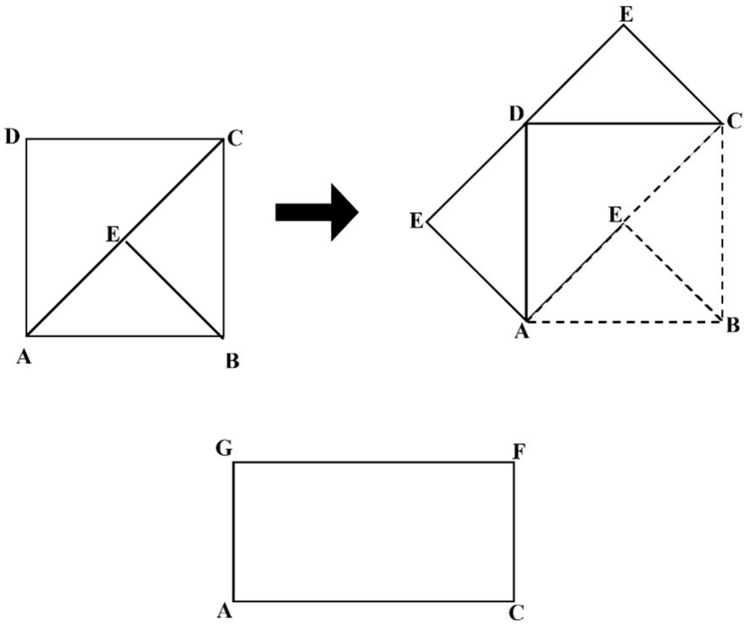

Figure 1.

Now, let us consider that the side of the square be 'x' units, then diagonal

$$Ac = \sqrt{2}x$$

Also, $BE = \frac{1}{2}AC = \frac{1}{2}\sqrt{2}x$

Then, according to the above construction length and breadth of rectangle ACFG,

$$AC = \sqrt{2}x \ \& \ AG = \frac{1}{2}\sqrt{2}x$$

Now, $ar(ABCD) = x^2$ and $ar(ACGF) = \sqrt{2}x \times \frac{1}{2}\sqrt{2}x \Longrightarrow ar(ACGF) = x^2$

So, $ar(ABCD) = ar(ACGF)$.

4.2. Transformation of a rectangle into a square of equal area

दीर्घचतुरश्रं समचतुरश्रं चिकीर्षंस्तिर्यङ्मानी करणीं कृत्वा शेषं द्वेधा वभिज्य पार्श्वयोरुपद ध्यात् । खंडमावापेन तत्संपूरयेत्तस्य नहिरि उक्तः ।। (BSS. 1-54)
To transform a rectangle into a square Baudhāyana suggests dividing the rectangle ABCD into two equal halves by joining a line parallel to its breadth MN. Now, again divide its other half (at south) by joining line PQ. Eliminate its one half from the south and place it on the eastern side of the first half. Complete the square by placing a small square in the corner. Take the eastern corner of this square as the center make an arc equal to the side of the square on the eastern side of the initial rectangle. The line AX which has been cut off by the arc on the length of the rectangle is the side of the desired square AXYZ (as shown in figure 2).

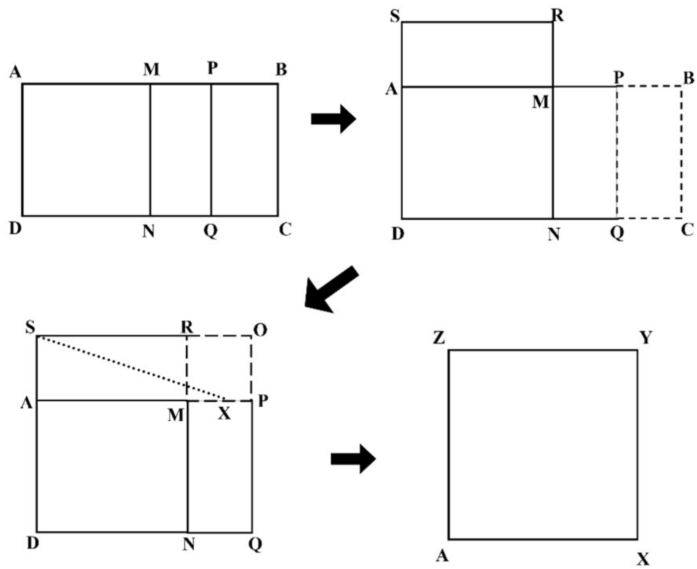

Figure 2.

Let

$$AB = a \ \& \ BC = b = \frac{a}{2} \Longrightarrow ar(ABCD) = ab.$$

Now, In rt.$\triangle SXA$,

$$AX^2 = SX^2 - SA^2$$
$$\implies AX^2 = SO^2 - BP^2$$
$$\implies AX^2 = (\frac{a}{2} + \frac{a}{4})^2 - (\frac{a}{4})^2$$
$$\implies AX^2 = \frac{a^2}{2} = ab$$

Also, $ar(AXYZ) = AX^2$.

Thus, we get ar(AXYZ)=ar(ABCD)

4.3. Transformation of a square into a circle of equal area

चतुरश्रं मण्डलं चकीर्षन्नक्ष्णायार्धमध्यात्प्राचीमभ्यापातयेत् ।यदतशिष्यिते तस्य सह मण्डलं परलिखिेत् ।। (BSS. 1-58)

To transform a square into a circle Baudhāyana suggests stretching a cord from the center of square PQRS to any one of its four vertices towards the prāci line. Now, cut this half diagonal equal to the length of half of its eastern side. A piece of cord will obviously remain between this new cut and the vertex of the square, divide this piece into three equal parts and describe a circle with the third part of the cord (as shown in figure 3).

Let $PQ = 2a$. Then, $ar(PQRS) = 4a^2$ ......(1)

Now, let the radius of the constructed circle be 'r'.

Then, $r = OX = ON + NX$

$$\implies r = PM + \frac{1}{3}(OQ - ON) = a + \frac{1}{3}(a\sqrt{2} - a) = \frac{1}{3}a(2 + \sqrt{2})$$

Now area of circle is πr^2, using the value of $^\circ$ and put $\sqrt{2} = 1.414214$ and $\pi = 3.141593$, we get,

Area of circle = 4.06901... $\times\, a^2$ ......(2)

Now, equation (2) is within about 1.7 % correct value of equation (1).

4.4. Transformation of a circle into a square of equal area

मण्डलं चतुरश्रं चकीर्षन्वष्किम्भमष्टौ भागान्कृत्वा भागमेकोन–त्रंशिघा वभिज्याष्टावंशितभिागानुद्धरेत् भागस्य च षष्ठमष्ठमभा–गोनम् ।। (BSS. 1-59)

To transform a circle into a square Baudhāyana suggests, dividing the diameter of the given circle into 8 parts. Further, split one of these eight parts into 29 parts and subtract 28 parts of these, as well as the sixth part of the proceeding sub-division less the eighth part of the last (as shown in figure 4).

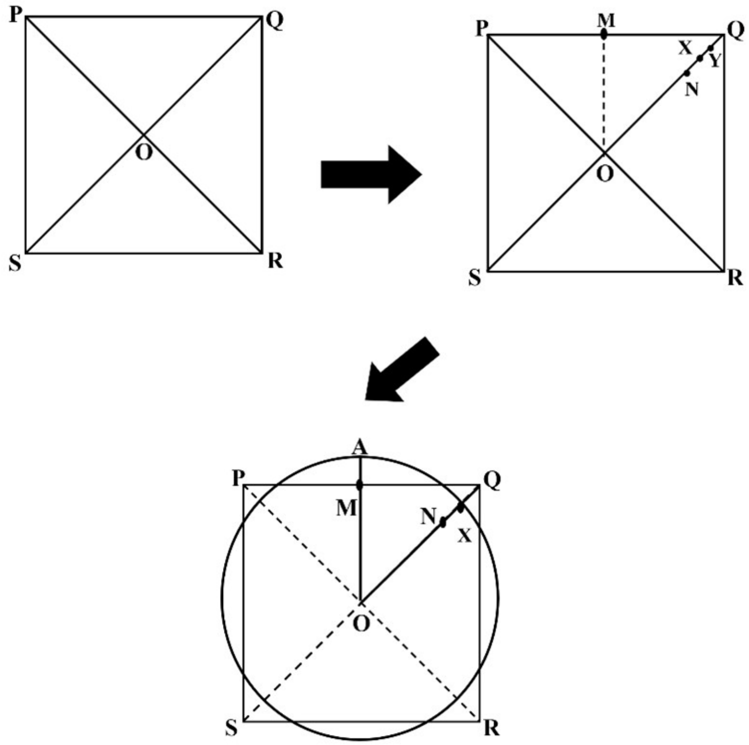

Figure 3.

Let d be the diameter of the given circle. Then, Baudhāyana says that the length of the desired square is,

$$length = d - \frac{d}{8} + \frac{d}{8 \times 29} - \frac{d}{8 \times 29 \times 6} + \frac{d}{8 \times 29 \times 6 \times 8} = \frac{9785}{11136}d.$$

Now, taking $\pi = 3.141593$, we get

$$\frac{Area of Square}{Area of Circle} = \frac{(\frac{9785}{1136})d^2}{\frac{\pi}{4}d^2} = 0.983045... \approx 1.$$

This result is approximately accurate up to 1.7 %.

5. Construction of some citi

As we discussed above, every citi is a low platform consisting of five layers of bricks such that odd layers are the same design, as are evens.

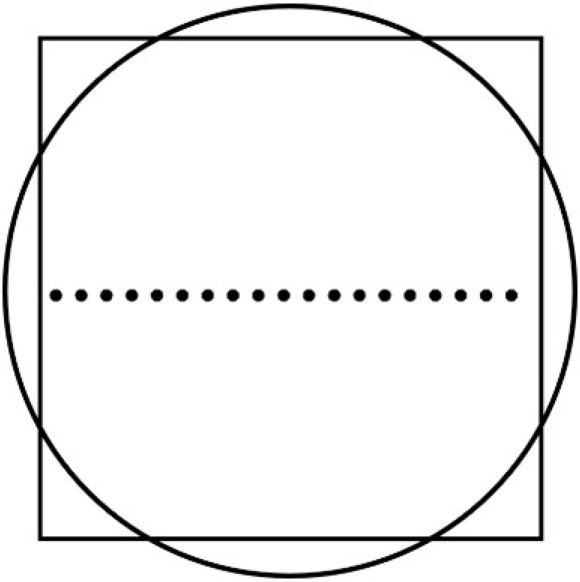

Figure 4.

5.1. Construction of Vakrapakśaśyenacit (वक्रपक्षोचितिं)

अथ वै भवतिश्येनचितिं चिन्वीत सुवर्गकाम इति। अथ वक्रपक्षो व्यस्तपुच्छ ।। (BSS. 3-1, 4-1)

The one who desires heaven may construct the altar in the shape of a Falcon. Construct the falcon with curved wings and outspread tail.

Size and shape of the bricks

Five types of bricks of different shapes and sizes are used here.

तस्येष्टकाः कारयेत्पुरूषस्य चतुर्ध्यः ।। (BSS. 4-2)

For construction purposes square bricks of side 30 angulas are to be made called Caturthī-bricks.

तासामर्ध्याः पाद्याश्च । अर्धपदेन पदेनाध्यर्धपदेन पदसविशेषेणेति।। (BSS. 4-3,6)

Also forms the bricks equal to half of caturathis (Ardhyā-brick) and a quarter of Caturthi (Padyā-brick), with half of a pada, one and a half pada, and the savisesa of a pada (Trapezium Pādyā-brick).

ते द्वे यथा दीर्घसणश्लिष्टे स्यातां तथार्धेष्टका कारयेत् ।। (BSS. 4-7)

Put together two four-cornered quarter-bricks with their long sides to form a half-brick (Hansamukhi-brick).

Arrangement of bricks in the odd layers

उपधाने शरसोप्यये चतुर्थीमुपद्धयात् । हंसमुखी पुरस्तात् ।। (BSS. 4-26, 27)

A caturthi is to be placed at the joint of the head and atman (body). Place quarter bricks on both sides of hansmukhi.

पादेष्टके अभतिः । तयोरवस्तादभतिस्तशि्रस्तशि्रश्चतुरश्रपाद्याः ।। (BSS. 4-28, 29)

A hansmukhi is placed on the east of the caturthi. To the west of caturthi and hansmukhi, place two triangular quarter bricks on both sides.

शेषे पादेष्टकाः ।। (BSS. 4-30)

Place quarter bricks in the remainder of the head.

अपिवा शरिसो अग्रेहंसमुखीमुपदध्यात् तस्या अवस्ताच्चतुर्थी– मुपदध्यात् पादेष्टके अत–स्ययोरवस्तादभतिः तशि्रस्तशि्रश्च–तुरश्रपाद्याः शेषे पादष्टकाः ।। (BSS. 4-31)

Place a hansmukhi brick in the top of the head. Place a caturthi in the west, a triangular quarter brick on each side of it. On the west of each quarter brick place 3 four cornered quarter bricks. Cover the remaining area with triangular quarter bricks.

शरिसोवस्तात्प च पादेष्टका व्यतषिक्ता उपदध्यात् । तथा पुच्छस्या पुरस्तात् ।। (BSS. 4-32, 33)

Place 5 triangular quarter bricks mutually joined one another, to the west of the head. Repeat the same process to the east of the tail.

यद्यदपच्छन्निं तस्मन्निर्धेष्टकाः पादेष्टकाश्चोपदघ्यात् । शेषमग्नंिचतुर्भागीयाभिः प्रच्छादयेत् ।। (BSS. 4-34, 35)

Place half bricks and quarter bricks there, where something has been cut off. Cover the remainder of agni with chaturthi bricks.

पाद्याभिः सार्ध्याभिः संख्या पूरयेत् ।। (BSS. 4-36)

Finally, complete the number of 200 bricks with quarter and half bricks.

Arrangement of bricks in the even layers

अपरस्मन्प्रिस्तारे हंसमुखीश्चतश्रश्चसृमिः पादेष्टकाभिः संजोजयेद्यथा दीर्घचतुरश्र संपद्यते तत्तर्मिक् स्वयमातृण्णावकाश उपदध्यात् ।। (BSS. 4-37)

Combine 4 hansmukhis and 4 triangular quarter bricks to form an oblong; place this oblong side-ways on the spot of the svayamatranna.

हंसमुख्यौ प्रतच्यिौ पुच्छाप्यये अर्धपदेनात्मनविशिये । तयोरवस्तादभतिस्तस्ति्रः पादेष्टकाः प्रायग्रमुखीरूपदघ्यात् ।। (BSS. 4-38, 39)

Place two hansamukhi turned toward the west there, where body and tail are joined. Place 3 triangular quarter bricks tops towards the east on the west of these hansamukhis.

पुच्छस्यावस्तात्पञ्चदश पादेष्चका व्यतष्क्ति उपदध्यात् । पादेष्टके अर्धेष्टकेतपिक्षपत्राणां प्राचीत्र्यत्यासं चनियात् ।। (BSS. 4-40, 41)

Place 15 triangular quarter bricks joined together at the west of the tail. Place 2 triangular quarter bricks and 1 half brick in the patras of the wings, towards east from the west.

वशिये यदपच्छनिं तस्मन्निर्धेष्टकाः पादेष्टकाश्चोपदघ्यात् । शेषमग्नं चतुर्भागीयाभिः प्रच्छादयेत्पाद्याभिः सार्ध्याभिः संख्यां पूरयेत् ।। (BSS. 4-42, 43)

Place half bricks and quarter bricks at the place where the body joined to the wings and those places where something is cut off. Cover the rest agni with chaturthis and complete the number of 200 bricks with quarter and half bricks.

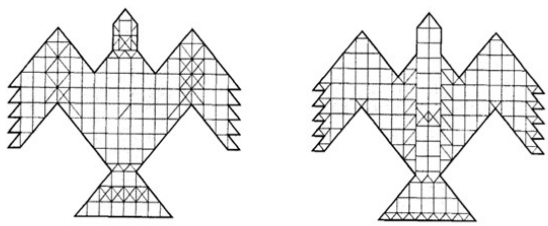

Figure 5. Odd layer & Even Layer

5.2. Construction of Rathacakracit (रथचक्रचतिं)

रथचक्रचतिं चन्वीतेतविज्ञायते ।। (BSS. 5-1)

The sacrificer is to construct the agni or altar in the shape of chariot-wheel, such is in the tradition.

Shape and Size of the Bricks

Only one kind of brick is used here.

पुरूषार्धात्पंचदशेनेष्टकाः समचतुरश्राः कारयेन्मानार्थाः ।। (BSS. 5-10) Square bricks of length equal to the fifteenth part of half of puruṣa were to be made which are used to measure the area of citi only; not for construction of citi.

तासांदेशते पंचवण्शितशिच सारत्नप्रिादेशः सप्तवधिः संपद्यते ।। (BSS. 5-11)

225 such bricks together constitute the seven-fold agni.

(480 ×225 = 10800 sq. angulas = $7\frac{1}{2}$ sq. purusa)

Arrangement of the bricks in the odd layers

तास्वन्याः चतुःषष्टमिावपेत् ।। (BSS. 5-12)

Add 64 more bricks to these 225 bricks.

ताभःिसमचतुरश्रं करोती । तस्य षोडशेष्टका पार्श्वमानी भवत ि।। (BSS. 5-13) With all these bricks a square is to be formed and each edge of the square is compromises 16 bricks.

त्रयस्त्रर्ंशिदतशिष्यिन्ते ।। (BSS. 5-15)

After forming a square of sixteen by sixteen, 33 bricks were still remains.

ताभरिन्तान्सर्वतः परचिनिुयात् ।। (BSS. 5-16)

Place all the remaining bricks on all the sides of the square to round the borders.

नाभः िषोडश मध्यमाः ।। (BSS. 5-17)

16 middle bricks, forms the nave of the wheel.

(Area of nave = $16 \times 480 = 7680$ sq. angulas)

चतुः षष्टरिराश्चतुः षण्टर्विेदः ि।। (BSS. 5-18)

64 bricks form spokes of wheels and these 64 also form vedi i.e., the empty space between two spokes.

नेमः िशेषाः ।। (BSS. 5-19)

The remaining (145) bricks form the felloe of the wheel.

नाभमिन्ततः परलिखिेत् । नेममिन्ततश्चान्तरतश्च परलिख्यिा ।। (BSS. 5-20, 21)

Transform the square nave into a circle and also turn the inner and outer square edges of felloe, into circles.

नेम िनाभ्योरन्तराल दूआत्रणि्शधा वभिज्य वपिर्यास भागानुद्वरेत् ।। (BSS. 5-22)

Now, divide the area between the nave and the felloe into 32 parts and take off the 2^{nd}, 4^{th}, 6^{th}, 8^{th}, etc., parts.

एवमावाप उद्धृतो भवती ।। (BSS. 5-23)

In this manner, the extra area of 64 bricks is removed.

नेमचिंतुः षष्टं िकृत्वा व्ययवलख्यि मध्ये परकृिषेत् । ता अष्टावशंि–तशितं ।। (BSS. 5-24, 25)

Describing a circle in the middle of the felloe, by drawing the separating lines divide the felloe into 64 parts. Then we get 128 bricks placed in the felloe.

अरांश्चतुर्धा विभजेत् । नाभ मष्टधा विभजेत् ।। (BSS. 5-26, 27)

Divide each spoke into four parts and the nave into eight parts. (There were a total of 200 bricks in the first layer, 128 in the felloe, $16 \times 4 = 64$ bricks in spokes, and 8 in the nave.)

Arrangement of the bricks in the even layers

अपरस्मिन् प्रस्तारे । नाभमिन्ततश्चतुर्थवेलायां परकृिषेत्त ।। नेममिन्तरतः (BSS. 5-28, 29, 30)

In the second layer, at a distance of a quarter from the edge a circle is to be described in the nave and also in felloe from its inner edge.

नेममिन्तरतश्चतुः षण्टिं कृत्वा व्यवलखित् । अराणां पञ्चधा वभिागं आपरकिर्षणयोः ।। (BSS. 5-31, 32)

After dividing the felloe into 64 parts, draw dividing lines. Further divide the spokes into five equal parts, each up to the two circles i.e., nemi and nābhi.

नेम्यामन्तरालेषु देऊ देऊ । नाम्यामन्तरालेष्वेकैकाम् ।। (BSS. 5-33, 34)

Place two bricks in each of the interstices in the felloe. Place one brick in each of the interstices of the nave.

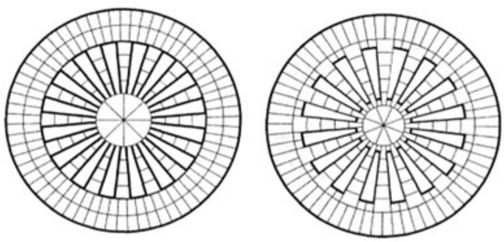

Figure 6. Odd layer & Even Layer

यच्छेषं नाभेस्तदष्टधा वभिजेत् । स एष षोडशकरणः सारो रथचक्रचति् ।। (BSS. 5-35, 36)

Divide the remainder of the nave into eight parts. This is the required construction for the shape of a wheel with spokes, which requires 16 different kinds of bricks altogether.

5.3. Construction of Kūrmacit (कूर्मचतिं)

कूर्मचतिं चन्वीत यः कामयेत ब्रह्मलोकमभजियमतविज्ञायते ।। (BSS. 9-1)

The sacrificer who wishes to conquer the world of Brahmna has to construct the agni in the shape of a turtle; such is in the tradition.

द्वयाः खलुकूर्मा भवन्तविक्राङ्गाश्च परमिण्डलाश्च ।। (BSS. 9-2)

There are two types of tortoises, one with angular limbs and the other with round limbs.

Shape and size of the Agni

अथाग्नंविमिमीते । चतुरश्रं आत्मा भवति। तस्य दश प्रक्रमाः पार्श्वमानी भवति।। (BSS. 9-3, 4)

The agnikshetra for this citi is a square and its side is 300 angula.

तस्य द्वाभ्यां द्वाभ्यां प्राक्रमाभ्यांश्रक्तीनामपच्छेदः । पूर्वस्मन्निनकें प्रक्रमप्रमाणानि चत्वारिचतुरश्राणिकृत्वा तेषां ये अन्त्ये ते अक्ष्णयापच्छन्दि्यात् ।। (BSS. 9-5, 6)

Cut off the corners of this square, with 60 angula each. On the middle of the eastern side, make four squares of side 30 angula each. Such that, one divides the two by the diagonal and removes half of each square.

एवं दक्षणत् एवं पश्चादेवमुत्तरतः । स आत्मा ।। (BSS. 9-7, 8)

The same process is to be done on the southern, western, and northern sides. The atman is formed.

शरः पञ्चपदायाममर्धपुरुषव्यासम् । तस्यां सौ प्रक्रमेण–प्रक्रमेणा पच्छिन्द्यात् ।। (BSS. 9-9, 10)
The head of the turtle is 75 angula long and 60 angula wide. Cut off two east corners of the head of width 30 angula each.

श्रक्त्यपच्छेदे पादानुन्नयेत् । तस्य द्विपदाक्ष्णयां तरशच तद्विगुणायाममेनूची ।। (BSS. 9-11, 12)

From where the corners have been cut off, the feet of the turtle are to be lengthened out. The breadth of one foot is equal to the diagonal of a square of side=30 angula = 2 pada and the length is double of its breadth.

तस्या द्वपिदाक्ष्णया पूर्वमण्समपच्छन्दि्यात् । एतेनेतरेषां पादानाम पच्छेदा व्याख्यातः ।। (BSS. 9-13, 14)

To create the rectangle, cut off the eastern corner equal to the diagonal of the square of side 30 angula by drawing a line from the mid-point of north-east side to the southern corner. Cutting off of a corner of each, other feet would explain.

अपरयोः पादयोरपरावंसावपच्छन्दि्यात् । एवं सारत्नप्रिादेशः सप्तवधिः संपद्यते ।। (BSS. 9-15, 16)

For two western feet cut off the western corners. By this, we brought about seven-fold agni with the two aratni and the pradesa.

Shape and size of the Bricks

तस्येष्टकाः कारयेत्पुरूषस्य चतुर्थ्यः । तासामध्र्याः पाद्याश्च ।। (BSS. 9-17)

For this citi, square bricks of side 30 angula are to be made. Half and a quarter of these bricks are also made by drawing diagonals.

अध्यर्धापाद्याश्चतुर्भिः परिगृहीयात्प्रक्रमेण द्वाभ्यां पदाभ्यां पदसविशेषेणेति ।। (BSS. 9-18)

The brick which is the fourth part of a brick equal to one chaturthi (a square of side 30 angula) and a half is enclosed with four sides, one of 30 angula long, two of 15 angula long and one equal to savisesa of 15.

[Area of Pādyā brick = $\frac{1}{4}900$ = 225 sq.angulas, area of Ardhyārdha of Pādyā brick = 225 + $\frac{1}{2}225$ sq. angula, & area of trapezium Pādyā brick = $15 \times 15 + \frac{1}{2}(15 \times 15) = 225 + \frac{1}{2}225$ sq. angula]

ते देदू यथे दीर्धसं श्लष्टे स्यातां तथैकां कारयेत् । द्विपदाक्ष्ण्यार्धेन समचतुरश्रामेकाम् ।। (BSS. 9-19, 20)

One brick is formed by joining two of these bricks with their long edge. A square brick is to be made by taking the side equal to the diagonal of the square of side 2 pada.

Arrangement of the bricks in the odd layers

उपधाने शरसोडग्रे चतुरश्रामुपदध्यात् । हंसमुख्यावावस्तात ।। (BSS. 9-21, 22)

A square brick is to be placed at the top of the head of the turtle and two hansamukhi to the west of it.

पञ्चपञ्च चतुरश्रा देदूदेदू पादेष्टके इति पादेषूपदध्यात् । यद्यदपच्छन्निं तस्मन्निर्धेष्टका उपदध्यात् ।। (BSS. 9-23, 24)

Place 5 square and 2 quarter bricks in each foot and place half bricks to the piece cut off from the agniksetra.

शेषमग्नं चतुर्भागीयाभिः प्रच्छादयेत् । अर्धेष्टकाभिः संख्र्यां पूर्येत ।। (BSS. 9-25, 26)

Cover the remaining part of the agni with caturthis. The number (200) is to be filled with half bricks.

Arrangement of the bricks in the even layers

अपरस्मन्प्रिस्तारे शरिसोयग्रे हंसमुखीमुपदध्यात्पादेष्टके अभतिः । तयोरवस्तादभतिो देदू–देदू अध्यर्धापाद्ये वषिूची ।। (BSS. 9-27, 28)

Place a hansamukhi brick in the top of the head and two quarter bricks on each side of the head. On the west of these quarter bricks place 4 bricks equal to quarter of an adhyardha, turned towards different directions (tops of two is in east and others in west).

तयोरवस्तादभतिरछेदसंहति देदू पादेष्टके । देदू–देदू द्वपिदे तशि्रस्तशि्रोयर्धेष्टकाइति पादेषूपदध्यात् ।। (BSS. 9-29, 30)

A triangular quarter brick with the cut is connected to the west of these on each side. Two dwipada bricks and three half bricks are to be placed in every foot.

यद्यदपच्छन्निं तस्मन्निर्धेष्टकाः पादेष्टकाश्चोपदध्यात् । शेषमग्निं चतुर्भागीयाभिः प्रच्छादयेत् ।। (BSS. 9-31, 32)

The half and the quarter bricks are to be placed on the cut-offs on the edges of the agniksetra. Cover the remaining part of the agni with caturthi bricks.

अर्धेष्टकाभः संख्यां पूर्येत ।। (BSS. 9-32)

And finally, the number (200) is to be filled with half bricks.

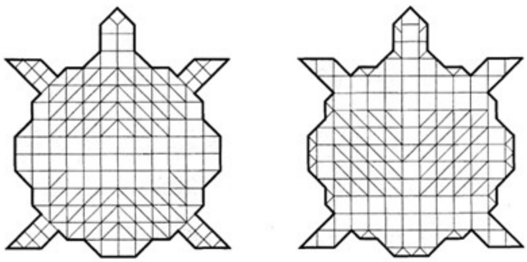

Figure 7. Odd layer & Even Layer

6. Conclusion

It appears that ancient Indians demonstrated proficiency in performing essential numerical calculations and practical geometric patterns, particularly evident in the context of the Indus Valley Urban civilizations. The Vedic period witnessed a zenith in the development of mathematics. It is evident that various branches of ancient mathematics, including algebra, arithmetic, and geometry, were not only further developed but also cultivated by Vedic Indian mathematicians. These mathematical advancements found applications in diverse fields such as astronomy, prosody, and ritualistic practices during that era. It is clear that the area of each layer of citi is same and equal to $7\frac{1}{2}$ sq. puruṣa. Further it is also clear that the bricks are chosen in such a manner that they are 200 in numbers in each layer and covers the area of $7\frac{1}{2}$ sq. puruṣa. Application of construction involved in square to circle transformation to the constructions of citi having circular shapes as Ratha-Cakracit (रथचक्रचतिं) and the methods involved in expansion of citi by scaling up all the dimensions; shows the great level of integrated wholeness, connectedness and consistency of the body of results achieved by Śulba Sūtras.

Author contributions:

Literature Survey, develop theoretical framework, primary analysis, manuscript drafting, reviewing, validation: **Vishal Kumar**; *Supervising, data interpretation, reviewing, validation:* **Ajendra Kumar**

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References

[1] T.A.S. Amma, *Geometry in ancient and medieval India*, Motilal Banarsidass Publication, Delhi, India (1999).
[2] L. V. Gurjar, *Ancient Indian Mathematics and Vedha*, Ideal Book Services, Poona, India (1947).
[3] R. P. Kulkarni, *Geometry according to śulba sutra*, Vaidka Samshodhan Mandala, Pune, India (1983).
[4] S. P. Saraswati, *Founders of sciences in ancient India*, Govindram Hasanand, Delhi, India (1986).
[5] S. Sharma, सद्धिान्तशरिोमणिः, Chaukhamba Surbharti Prakashan, Varanasi, India (2007).
[6] M. Winternitz, *The History of Indian Literature*, University of Culcutta, India (1927).
[7] A. K. Bag, *Ritual Geometry in India and its Parallelism in other Cultural areas*, Indian Journal of History of Science **25(1-4)** (1990) 4–19.
[8] B. Datta and A. S. Narayana, *Hindu geometry*, Indian Journal of History of Science Calcutta **15(2)** (1980) 88–-121.
[9] R. C. Gupta, *Śulba Sūtra: Earliest studies and a newly published manual*, Indian Journal of History of Science **41(3)** (2006) 317–320.
[10] P. Taneja and N. Handa, *Enlargement of vedis in the śulba sutra*, Indian Journal of History of Science **45(2)** (2010) 175–188.

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