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AN ALPHA-SERIES PROCESS OPTIMAL REPLACEMENT PROBLEM FOR A SHOCK MODEL WITH TWO-TYPE FAILURES

DR. RAJA SEKHARA REDDY K ✉

Abstract

In this paper, a shock model for a repairable system with two types of faults is investigated. It is found that two types of shock models in a sequence of random shocks cause the system to fail: one based on the interarrival time between two consecutive shocks being smaller than a given positive value δ, and the other based on the shock size of a single shock being larger than a given positive value δ. Let us further assume that the system is no longer "as good as new" after repair, while the consecutive operating times of the system become longer and longer. Under these assumptions, using the α-series process repair model, the replacement policy N is determined based on the number of system failures. The problem is to find an optimal replacement policy N^* so that the long-term average cost per unit time is the lowest. The explicit equation for the long-term average cost per unit time is derived so that an optimal policy N^* can be determined analytically and numerically. Finally, a numerical example is given to substantiate the theoretical results.

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1. Introduction

In most repair-replace models, a failed system is assumed to result in a “good as new" functional system after repair, and repair times are neglected so that successive operating times form a renewal process. This type of model can be described as a "perfect repair model".

Barlow and Hunter [1] studied a minimal repair model in which a minimal repair does not change the age of the system. Thereafter, an imperfect repair model was developed by Brown and Proschan [2], under which a repair with probability ‘p’ is considered as a perfect repair and with probability ‘**1-p**’ as a minimal repair. Many others worked in this direction and developed corresponding optimal replacement policies such as Block et al [7–9], Park [3], Finkelstein [4], Stadje and Zukerman [5], A.D.J. Stanley [6], and Yuan Lin Zhang and Guan Jun Wang [10–12].

In general, a deteriorating system can be assumed to have stochastically decreasing successive working times, while stochastically increasing successive repair times after failures due to aging and cumulative wear and tear in many systems. To model such a simple repairable deteriorating system, a repair may occur due to either internal shock or external shock.

The shock model is one of the important problems in reliability and maintenance theory. It is mainly used to study the external causes such as environmental factors that may influence a system. For example, in a power generator, a shock may be affected by random causes or by the operation of other instruments or electrical machinery.

Guan Jun Wang and Yuan Lin Zhang [13] considered a shock model for a repairable system with two types of failures by assuming that two types of shocks in a sequence of random shocks cause the system to fail: one based on the time interval between two consecutive shocks, which is smaller than a given positive value d, and the other based on the shock size of a single shock, which is larger than a given positive value (Ψ). Under this assumption, they obtained some reliability indices of the shock model, such as system reliability and the average working time before system failure. Moreover, the replacement policy N was determined based on the number of system failures by minimizing the long-term average cost per unit time.

Yuan Lin Zhang [19] studied a deteriorating repairable system with three states, including two fault states and one working state. A replacement policy N based on the number of failures of the system is assumed, where the system is replaced at the time of the N-th failure. An optimal replacement policy N^* is determined at which the average cost rate (ACR) is the lowest, and an explicit expression for the ACR is derived.

Lam et al. [18] developed a monotonic process model for a degenerative single-component system with $K + 1$ states (K failure states and one working state) and showed that this model corresponds to a geometric process model of a single-component system with two states, so that both systems have the same average cost rate and the same optimal strategy.

For a deteriorating system, a more reasonable repair model is the geometric process repair model first proposed by Lam Yeh [16, 17]. Under this model, Lam explained two kinds of replacement policy: one based on the working age T of the system and the other based on the failure number N of the system. The objective is to choose optimal replacement policies T^* and N^*, respectively, such that the long-run average cost per unit time is minimized. The explicit expressions for the long-run average cost per unit time under these two kinds of policies are derived, and the corresponding optimal replacement policies T^* and N^* are found analytically or numerically. Because the geometric process is a process that doesn't change direction and moves in one direction (monotone process).

Stadje and Zuckerman [5] studied a general monotonic process repair model to generalize Lam's work. Yuan Lin Zhang [19] generalized Lam's work through a bivariate replacement policy (T, N), where the system is replaced at T (working age) or N (Nth failure), whichever comes first. To improve system reliability or increase system availability, Yuan Lin Zhang [20] applied the geometric process repair model to a two-component cold standby system with one repairer. Assuming that each component is not "as good as new" after repair, he considered a replacement policy N based on the number of repairs of component 1 using the geometric process. The problem is to determine the optimal replacement policy N^* such that the long-term expected profit per unit time is maximized.

Brown et al. [21] explained some important properties of monotone processes and proved that alpha series processes are better suited to model uptimes. Based on this understanding, we proposed in this paper to develop a model for alpha series process maintenance and find an optimal replacement policy.

In this paper, a shock model for a repairable system with two types of faults is investigated. It is assumed that two types of shocks in a sequence of random shocks cause the system to fail: one based on the time between two consecutive shocks being smaller than a given positive value δ ("δ-shock model"), and the other based on the shock magnitude of a single shock being larger than a given positive value δ. Further, we assume that the system is not "as good as new" after repair, while the successive ramp-up times of the system become longer and longer. Under these assumptions, using the α-series process repair model, the replacement policy N is determined based on the number of system failures. The problem is to find an optimal replacement policy N^* so that the long-term average cost per unit time is minimized. The explicit equation for the long-run average cost per unit time is derived, and thus an optimal policy N^* can be determined analytically or numerically. Finally, a numerical example is given to support the theoretical results.

2. The Model

When a repairable system is working, it may fail due to an external cause such as a random shock. In this section, we consider a system where two kinds of random shocks occur, leading to system failure. Let T_n and D_n denote the time interval between the $(n-1)$-th shock and the n-th shock, and the magnitude of the n-th shock, respectively. The system will fail as soon as $T_n < \delta$ appears, $n = 1, 2, \ldots$, and the system failure caused by $T_n < \delta$ is called the first type of failure. In fact, it is a δ-shock model considered by Wang and Zhang (2001). Note that the system will fail as soon as both $T_n < \delta$ and $D_n > \gamma$ appear, $n = 1, 2, \ldots$. Thus, we may as well call the system failure as the third type failure. In other words, a failure of two types leads to three cases of system failure. It is a more extensive shock model in which we not only consider the magnitude of single shock but also the time interval of consecutive two shocks to the effect of the system.

Under this model, it is assumed that:

1. At the beginning, the system is new. The system will be repaired as soon as it fails. The system will be replaced some time by a new and identical one, and the replacement time is negligible.
2. Assume that the system after repair is not 'as good as new' while the consecutive repair times of the system form a stochastic increasing alpha process. The time interval between the completion of the $(n-1)$-th repair and the completion of the n-th repair on the system is called the n-th cycle of the system, $n = 1, 2, \ldots$.

3. Let X_n and $Y_n^{(i)}$ be respectively the working time and the repair time of the system incurred by the i-th type failure in the n-th cycle, $i = 1, 2, 3$; $n = 1, 2, \ldots$.
4. Let $\{X_n, n = 1, 2, \ldots\}$ be a sequence of i.i.d. non-negative random variables with distribution function $F(n^{\alpha} t)$, while the distribution of $Y_n^{(i)}$ is assumed to be $G(n^{\beta_i} t)$, and assume that $ET_1 = \lambda$ and $EY_1^{(i)} = \mu_i$, where $t \geq 0, \beta_i < 0, \alpha > 0, \lambda > 0, \mu_i > 0$ for $i = 1, 2, 3$; $n = 1, 2, \ldots$.
5. Assume that $X_n, Y_n^{(i)}$, $i = 1, 2, 3$; $n = 1, 2, \ldots$ are stochastically independent.
6. The replacement policy N, in which the system is replaced when the failure number of the system reaches N, is considered.
7. Assume that the repair cost incurred by i-th type failure of the system per unit time is $C_r^{(i)}$, $i = 1, 2, 3$, the working reward of the system per unit time is C_w, and the replacement cost of the system is C.

3. The Long-Run Average Cost Rate (ACR)

According to the renewal reward theorem (see Ross [22, 23]), let $S_1, S_2, \ldots$ be the shock arrival times when the system is working, and assume that $\{T_n, n = 1, 2, \ldots\}$ and $\{D_n, n = 1, 2, \ldots\}$ are respectively a sequence of i.i.d. non-negative random variables. Clearly,

$$S_n = T_1 + T_2 + \ldots + T_{n-1}, \quad n = 1, 2, \ldots \tag{3.1}$$

Let X be the working time before the system first fails, then

$$X = T_1 + T_2 + \ldots + T_M, \tag{3.2}$$

where M is the shock number of times at the system failure, and it is a random variable. Now, we give out some results of the system under the shock model with two-type failure. According to Definition 3, $\{T_n, n = 1, 2, \ldots\}$ and $\{D_n, n = 1, 2, \ldots\}$ are respectively a sequence of i.i.d. non-negative random variables. We might as well let T and D denote respectively the time interval of consecutive two shocks and the magnitude of a single shock.

Theorem 3.1. *Assume that $\{(T_n, D_n), n = 1, 2, \ldots\}$ forms a correlated renewal sequence pair with joint distribution function*

$$F(xy) = P(T_n \leq x, D_n \leq y), \tag{3.3}$$

and $H(t)$ is the distribution function of X, then the Laplace transform of $H(t)$ is given by

$$H^*(s) = \frac{F_T^*(s) - F^*(s, \gamma)}{1 + \int_0^{\delta} e^{-sx} dF(x, \gamma) - F^*(s, \gamma)}$$

$$\text{where } F_T^*(s) = \int_0^{\infty} e^{-sx} F_T(x) dx, \quad F_T^*(s, \gamma) = \int_0^{\infty} e^{-sx} dF_T(x, y),$$

δ and γ are two given positive parameters.

(See G. J. Wang & Y. L. Zhang [13])

Thus, we can get the system reliability $R(t) = \bar{H}(t)$ or its Laplace transform $R^*(s)$ from Theorem 3.1.

Let

$$P_{00} = P(T > \delta, D < \gamma) \tag{3.4}$$

$$P_{10} = P(T < \delta, D < \gamma) \tag{3.5}$$

$$P_{01} = P(T > \delta, D > \gamma) \tag{3.6}$$

and

$$P_{11} = P(T < \delta, D > \gamma) \tag{3.7}$$

denote respectively the probabilities that neither the first type failure nor the second type failure happens, only the first type failure happens, only the second type failure happens, and both types of failures happen in the system.

Clearly, $p_{00} + p_{10} + p_{01} + p_{11} = 1$.

THEOREM 3.2. *Under the condition of Theorem 3.1 given by G. J. Wang & Y. L. Zhang [13], the mean working time before the system fails is given by:*

$$EX = \frac{ET}{1 - p_{00}}. \tag{3.8}$$

We make the following **assumptions** about the repairable system for random shock with two-type failures.

To determine an optimal replacement policy N such that the long-run average cost rate per unit time is minimized, let τ_1 be the first replacement time and let $\tau_n (n > 1)$ be the time between the $(n-1)$-th replacement and the n-th replacement. Obviously, $\{\tau_1, \tau_2, \tau_3, \ldots\}$ forms a renewal process. Let $C(N)$ be the long-run average cost per unit time under the replacement policy N. Because $\{\tau_1, \tau_2, \tau_3, \ldots\}$ is a renewal process, the time interval between two consecutive replacements is a renewal cycle. Then according to the renewal reward theorem (see, for example, Ross [22, 23]), we have

$$C(N) = \frac{\text{expected cost incurred in a cycle}}{\text{expected length of a cycle}}. \tag{3.8.1}$$

where the length of the renewal cycle is given by

$$\begin{aligned} W &= X_1 + [Y_1^{(1)} I_1^{(1)} + Y_1^{(2)} I_1^{(2)} + Y_1^{(3)} I_1^{(3)}] + X_2 + [Y_2^{(1)} I_2^{(1)} + Y_2^{(2)} I_2^{(2)} + Y_2^{(3)} I_2^{(3)}] + \ldots \\ &+ X_N - 1 + [Y_N - 1^{(1)} I_N - 1^{(1)} + Y_N - 1^{(2)} I_N - 1^{(2)} + Y_N - 1^{(3)} I_N - 1^{(3)}] \\ &+ X_N \end{aligned} \tag{3.9}$$

To evaluate EW, we only need to calculate the following results by using the property of the conditional expectation:

$$E(Y_n^{(1)} I_n^{(1)}) = E[E(Y_n^{(1)} I_n^{(1)} | I_n^{(1)})] = P(I_n^{(1)} = 1) E(Y_n^{(1)}) \tag{3.10}$$

$$E(Y_n^{(2)}I_n^{(2)}) = E[E(Y_n^{(2)}I_n^{(2)}|I_n^{(2)})] = P(I_n^{(2)} = 1)E(Y_n^{(2)}) \tag{3.11}$$

$$E(Y_n^{(3)}I_n^{(3)}) = E[E(Y_n^{(3)}I_n^{(3)}|I_n^{(3)})] = P(I_n^{(3)} = 1)E(Y_n^{(3)}) \tag{3.12}$$

Let $q_i, i = 1, 2, 3$ be the probability of the i-th type failure when failure happens, then

$$E(I_n^{(1)}) = q_1 = P(I_n^{(1)} = 1) = \frac{P_{10}}{1 - P_{00}}, \tag{3.13}$$

$$E(I_n^{(2)}) = q_2 = P(I_n^{(2)} = 1) = \frac{P_{01}}{1 - P_{00}}, \tag{3.14}$$

$$E(I_n^{(3)}) = q_3 = P(I_n^{(3)} = 1) = \frac{P_{11}}{1 - P_{00}}. \tag{3.15}$$

Clearly, $q_1 + q_2 + q_3 = 1$; and
according to the assumption of the model, we have

$$E\left(Y_n^{(i)}\right) = \int_0^\infty y dG_n(n^\beta y) = \frac{\mu_i}{n^\beta}, i = 1, 2, 3. \tag{3.16}$$

$$E\left(X_n^{(i)}\right) = \int_0^\infty x dF_n(n^\beta x) = \frac{\lambda}{n^\beta}. \tag{3.17}$$

$$E\left(Y_n^{(1)}I_n^{(1)}\right) = \frac{\mu_1}{n_1^\beta} \cdot q_1 = \frac{\mu_1}{n_1^\beta}\frac{P_{10}}{1 - P_{00}} \tag{3.18}$$

$$E\left[Y_n^{(2)}I_n^{(2)}\right] = \frac{\mu_2}{n_2^\beta} \cdot q_2 = \frac{\mu_2}{n_2^\beta}\frac{P_{01}}{1 - P_{00}} \tag{3.19}$$

$$E\left[Y_n^{(3)}I_n^{(3)}\right] = \frac{\mu_3}{n_3^\beta} \cdot q_3 = \frac{\mu_3}{n_3^\beta}\frac{P_{11}}{1 - P_{00}} \tag{3.20}$$

According to the assumption (4) and equations 3.2, 3.8 of the model and using equations 3.16 to 3.20, the expected length of a renewal cycle is

$$EW = NEX + \mu_1 q_1 \sum_{n=1}^{N-1} \frac{1}{n^{\beta_1}} + \mu_2 q_2 \sum_{n=1}^{N-1} \frac{1}{n^{\beta_2}} + \mu_3 q_3 \sum_{n=1}^{N-1} \frac{1}{n^{\beta_3}} \tag{3.21}$$

$$= \frac{\lambda}{1 - p_{00}} \sum_{n=1}^{N} \frac{1}{n^\alpha} + \mu_1 \frac{p_{10}}{1 - p_{00}} \sum_{n=1}^{N-1} \frac{1}{n\beta_1} + \mu_2 \frac{p_{01}}{1 - p_{00}} \sum_{n=1}^{N-1} \frac{1}{n\beta_2} + \mu_3 \frac{p_{11}}{1 - p_{00}} \sum_{n=1}^{N-1} \frac{1}{n\beta_3} \tag{3.22}$$

According to equations 3.8 and 3.21, we have (

$$C(N) = \frac{c_r^{(1)}\mu_1 q_1 \sum_{n=1}^{N-1} E(Y_n^{(1)}) + c_r^{(2)}\mu_2 q_2 \sum_{n=1}^{N-1} E(Y_n^{(2)}) + c_r^{(3)}\mu_3 q_3 \sum_{n=1}^{N-1} E(Y_n^{(3)}) + C - c_w NEX}{EW}$$

According to equations 3.8 and 3.21, we have

$$C(N) = \frac{c_r^{(1)}\mu_1\left(\frac{p_{10}}{1-p_{00}}\right)l_1 + c_r^{(2)}\mu_2\left(\frac{p_{01}}{1-p_{00}}\right)l_2 + c_r^{(3)}\mu_3\left(\frac{p_{11}}{1-p_{00}}\right)l_3 + C - C_w\lambda\left(\frac{1}{1-p_{00}}\right)\sum_{n=1}^{N}\frac{1}{n^{\alpha}}}{\lambda\left(\frac{1}{1-p_{00}}\right)\sum_{n=1}^{N}\frac{1}{n^{\alpha}} + \mu_1\frac{p_{10}}{1-p_{00}}\sum_{n=1}^{N-1}\frac{1}{n_1^{\beta}} + \mu_2\frac{p_{01}}{1-p_{00}}\sum_{n=1}^{N-1}\frac{1}{n_2^{\beta}} + \mu_3\frac{p_{11}}{1-p_{00}}\sum_{n=1}^{N-1}\frac{1}{n_3^{\beta}}} \tag{3.23}$$

Where

$$l_1 = \sum_{n=1}^{N-1}\frac{1}{n_1^{\beta}}, \quad l_2 = \sum_{n=1}^{N-1}\frac{1}{n_2^{\beta}}, \quad l_3 = \sum_{n=1}^{N-1}\frac{1}{n_3^{\beta}}.$$

In the next section, numerical results are provided to highlight the obtained theoretical results, and also the obtained analytical results are presented graphically for easy evaluation and to draw the conclusions.

4. Numerical Results and Conclusions

For the given hypothetical values of the parameters μ_1, μ_2, μ_3, λ, $C_r^{(1)}$, $C_r^{(2)}$, $C_r^{(3)}$, q_1, q_2, q_3, β_1, β_2, β_3, C, and C_w, we determine the long run average cost per unit time from the equation (9) as follows: Where $\mu_1 = 50$, $\mu_2 = 60$, $\mu_3 = 70$, $\lambda = 30$, $C_r^{(1)} = 10$, $C_r^{(2)} = 20$, $C_r^{(3)} = 30$, $q_1 = 0.25$, $q_2 = 0.35$, $q_3 = 0.40$, $\beta_1 = -0.65$, $\beta_2 = -0.75$, $\beta_3 = -0.95$, $\alpha = 1.25$, $C = 45000$, $C_w = 30$.

N	C(N)	N	C(N)
1	34.17944	13	35.26192
2	26.34778	14	36.62301
3	23.49444	15	37.89297
4	**22.81611**	16	39.07360
5	23.27011	17	40.16857
6	24.35011	18	41.18262
7	25.77206	19	42.12097
8	27.36431	20	42.98903
9	29.01965	21	43.79216
10	30.67077	22	44.53551
11	32.27607	23	45.22395
12	33.81093	24	45.86204

Table 1. The long-run average cost rate (ACR)

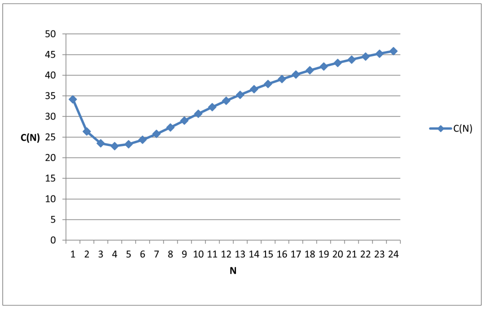

FIGURE 1. The long-run average cost rate (ACR) values Against N

For the given hypothetical values of the parameters μ_1, μ_2, μ_3, λ, $C_r^{(1)}$, $C_r^{(2)}$, $C_r^{(3)}$, q_1, q_2, q_3, β_1, β_2, β_3, C, and C_w, we determine the long run average cost per unit time from the equation (9) as follows: Where $\mu_1 = 50$, $\mu_2 = 60$, $\mu_3 = 70$, $\lambda = 30$, $C_r^{(1)} = 10$, $C_r^{(2)} = 20$, $C_r^{(3)} = 30$, $q_1 = 0.25$, $q_2 = 0.35$, $q_3 = 0.40$, $\beta_1 = -0.65$, $\beta_2 = -0.75$, $\beta_3 = -0.95$, $\alpha = 1.005$, $C = 45000$, $C_w = 30$.

N	C(N)	N	C(N)
1	30.94727	13	28.43450
2	21.86832	14	29.89445
3	18.28024	15	31.28317
4	**17.08297**	16	32.59772
5	17.14322	17	33.83775
6	17.92153	18	35.00462
7	19.11916	19	36.10080
8	20.55542	20	37.12943
9	22.11544	21	38.09404
10	23.72441	22	38.99835
11	25.33324	23	39.84609
12	26.91001	24	40.64094

TABLE 2. The Values Long-Run Average Cost Rate (ACR)

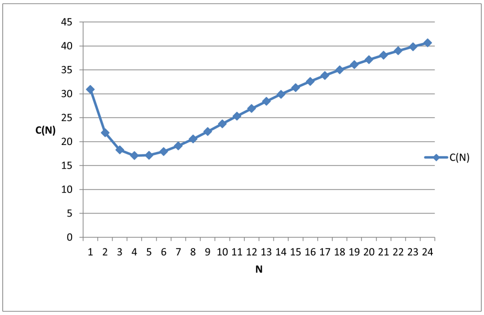

FIGURE 2. The long-run average cost rate (ACR) values Against N

For the given hypothetical values of the parameters μ_1, μ_2, μ_3, λ, $C_r^{(1)}$, $C_r^{(2)}$, $C_r^{(3)}$, q_1, q_2, q_3, β_1, β_2, β_3, C, and C_w, we determine the long run average cost per unit time from the equation (9) as follows: Where $\mu_1 = 50$, $\mu_2 = 60$, $\mu_3 = 70$, $\lambda = 30$, $C_r^{(1)} = 10$, $C_r^{(2)} = 20$, $C_r^{(3)} = 30$, $q_1 = 0.25$, $q_2 = 0.35$, $q_3 = 0.40$, $\beta_1 = -0.65$, $\beta_2 = -0.75$, $\beta_3 = -0.95$, $\alpha = 0.95$, $C = 45000$, $C_w = 30$.

N	C(N)	N	C(N)
1	30.19305	13	26.70477
2	20.83223	14	28.17112
3	17.07673	15	29.57233
4	15.75637	16	30.90444
5	**15.71734**	17	32.16618
6	16.41338	18	33.35808
7	17.54337	19	34.48188
8	18.92553	20	35.54014
9	20.44416	21	36.53584
10	22.02355	22	37.47227
11	23.61356	23	38.35279
12	25.18111	24	39.18077

TABLE 3. The Values Long-Run Average Cost Rate (ACR)

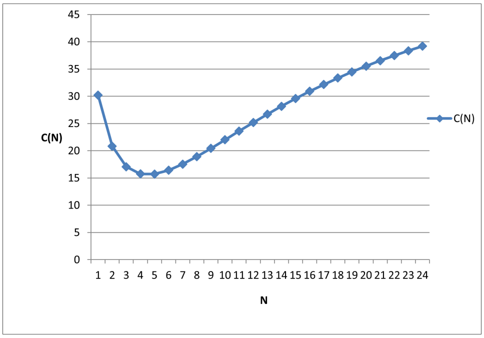

FIGURE 3. The long-run average cost rate (ACR) values Against N

For the given hypothetical values of the parameters μ_1, μ_2, μ_3, λ, $C_r^{(1)}$, $C_r^{(2)}$, $C_r^{(3)}$, q_1, q_2, q_3, β_1, β_2, β_3, C, and C_w, we determine the long run average cost per unit time from the equation (9) as follows: Where $\mu_1 = 50$, $\mu_2 = 60$, $\mu_3 = 70$, $\lambda = 30$, $C_r^{(1)} = 10$, $C_r^{(2)} = 20$, $C_r^{(3)} = 30$, $q_1 = 0.25$, $q_2 = 0.35$, $q_3 = 0.40$, $\beta_1 = -0.55$, $\beta_2 = -0.65$, $\beta_3 = -0.85$, $\alpha = 0.95$, $C = 45000$, $C_w = 30$.

N	C(N)	N	C(N)
1	30.19305	13	23.53606
2	20.72310	14	24.85165
3	16.75286	15	26.12913
4	15.14531	16	27.36205
5	**14.77744**	17	28.54661
6	15.12798	18	29.68087
7	15.91464	19	30.76423
8	16.96895	20	31.79706
9	18.18391	21	32.78036
10	19.48881	22	33.71562
11	20.83587	23	34.60458
12	22.19241	24	35.44922

TABLE 4. The Values Long-Run Average Cost Rate (ACR)

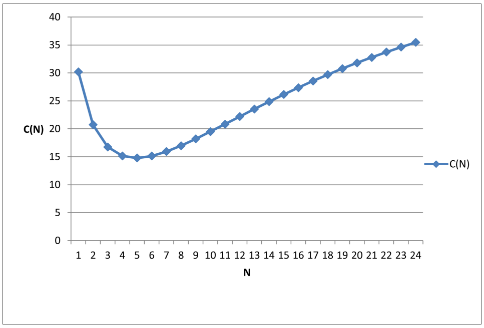

FIGURE 4. The long-run average cost rate (ACR) values Against N

5. Conclusion

1. From Table 1 and Figure 1, it is investigated that the long-run average cost per unit of time is minimized when the number of failures of the system reaches 4, i.e., $C(4) = 22.81611$ at $\beta_1 = -0.65, \beta_2 = -0.75, \beta_3 = -0.95$, and $\alpha = 1.25$. Thus, the system should be replaced at the time of the 4th failure.
2. From Table 2 and Figure 2, it is tested that the long-run average cost per unit of time is minimized when the number of failures of the system reaches 4, i.e., $C(4) = 17.08297$ at $\beta_1 = -0.65, \beta_2 = -0.75, \beta_3 = -0.95$, and $\alpha = 1.005$. Thus, the system should be replaced at the time of the 4th failure.
3. From Table 3 and Figure 3, it is identified that the long-run average cost per unit of time is minimized when the number of failures of the system reaches 5, i.e., $C(5) = 15.71734$ at $\beta_1 = -0.65, \beta_2 = -0.75, \beta_3 = -0.95$, and $\alpha = 0.95$. Thus, the system should be replaced at the time of the 5th failure.
4. From Table 4 and Figure 4, it is examined that the long-run average cost per unit of time is minimized when the number of failures of the system reaches 5, i.e., $C(5) = 14.77744$ at $\beta_1 = -0.55, \beta_2 = -0.65, \beta_3 = -0.85$, and $\alpha = 0.95$. Thus, the system should be replaced at the time of the 5th failure.
5. From the above conclusions (1) to (4), it is examined that the values of the parameters of the process are inversely related to the long-run average cost. This also coincides with practical analogy.

Author contributions:
Conceptualisation: Dr. Raja Sekhara Reddy K ; *Software:* Dr. Raja Sekhara Reddy K ; *Writing-Original Draft:* Dr. Raja Sekhara Reddy K

Conflicts of Interest: The authors declare no conflict of interest.

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Dr. RAJA SEKHARA REDDY K, Department of Mathematics, Government First Grade College, Gauribidanur, Karnataka, India
e-mail: manvirajreddy@gmail.com