

MAGNETO SORET INDUCED CONVECTION IN COUPLE STRESS NANOFLUID

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Abstract

The present study aims at investigating the combined effect of a uniform vertical magnetic field and the Soret effect on the onset of double diffusive convection in a couple stress nanofluid layer. The linear stability analysis is based on normal mode technique. Galerkin method has been applied to find the critical Rayleigh number and the corresponding wave number in terms of various parameters. The couple stress parameter has been found to stabilize the stationary convection couple stress is found to have destabilizing effect on oscillatory convection here. In this double diffusive convection under magnetic field, Darcy number also comes into play and has been observed to provide stabilizing effect on stationary and oscillatory modes. The effects of other different parameters on the stability of the system have been investigated analytically as well graphical analysis has been done. The comparison of results obtained has been done with the existing relevant studies.

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1. Introduction

Nanofluids are recent fluids that cause significantly enhanced thermophysical properties. Couple stress nanofluids have a significant importance in MHD power generators, for the arteries blockage-removal, cancer tumour treatment, hyperthermia etc. It is well-known that the flow field of a conducting fluid is altered on introducing a magnetic field. As far as the stability is concerned, the magnetic field, in general, is having a stabilizing effect apart from few exceptions.

Thermo-solutal convection in a couple stress fluid through a porous medium having vertical magnetic field and vertical rotation was studied by Kumar [1]. Rotation was observed to have a stabilizing effect but magnetic field and couple stress were observed to give both stabilizing and destabilizing effects. Instability of Magneto Hydrostatic stellar interiors from magnetic buoyancy were studied by Gilman [2]. Normal mode instability was observed due to magnetic buoyancy in fluids having large heat diffusivity compared with viscosity and magnetic diffusivity, as in stellar interiors. However, the magnetic buoyancy instability was found to be non-axisymmetric which is different from those in that, in a star with toroidal magnetic field. Schatzman [3] also did formal analysis on magnetic buoyancy instability but only for a special

case. Influence of magnetic field in couple stress fluids was studied by Shankar et al. [4]. They found that the effect of magnetic field is to slow down the onset of instability while an opposite kind of behaviour is observed with increasing couple stress parameter.

Sharma and Thakur [5] studied the effect of uniform magnetic field on convection in a couple stress fluid layer. The suspended particle effect in couple stress fluid layer heated from below was studied by Sharma and Sharma [6]. The magnetic field and couple stress both were found to have stabilizing and destabilizing effects in thermo-solutal convection problem for a couple stress studied by Kumar and Kumar [7]. Malashetty et al. [8] studied the Soret effect on double diffusive convection in a couple stress fluid and found significant effect of couple stress and destabilizing effect of positive Soret parameter. The literature survey indicates that no study has investigated the effect of magnetic field on double diffusive convection in a couple stress nanofluid layer with Soret factor. The present study examines the effect of vertical magnetic field on Soret induced double diffusive convection in a couple stress nanofluid horizontal layer.

2. Mathematical Formulation

An infinite isotropic porous layer of incompressible Maxwellian couple stress viscoelastic fluid confined between two horizontal planes has been considered, where the temperatures at the lower and upper boundaries are T_l^* and T_r^* respectively, T_l^* being greater than T_r^* . A uniform vertical magnetic field $\mathbf{M}^* = (0, 0, M_0^*)$ acts on the system. The governing equations are as follows:

$$\nabla^* \mathbf{q}_d^* = 0 \quad (2.1)$$

$$\begin{aligned} \frac{1}{K}(\mu - \mu_c \nabla^{*2}) \mathbf{q}_d^* &= \left(1 + \lambda^* \frac{\partial}{\partial t^*}\right) \left[-\nabla^* p^* + \left(\psi^* \rho_p + (1 - \psi^*) \left\{ \rho \left(1 - \beta_l (T^* - T_r^*) \right. \right. \right. \right. \\ &\quad \left. \left. \left. - \beta_c (S^* - S_r^*) \right\} \right\} \right) \mathbf{g} + \frac{\mu_e}{4\pi} (\nabla^* \times \mathbf{M}^*) \times \mathbf{M}^* \right] \end{aligned} \quad (2.2)$$

$$\begin{aligned} (\rho c)_M \frac{\partial T^*}{\partial t^*} + (\rho c)_F \mathbf{q}_d^* \cdot \nabla^* T^* &= k_m \nabla^{*2} T^* + \epsilon (\rho c)_P \left[B_d \nabla^* \psi^* \cdot \nabla^* T^* \right. \\ &\quad \left. + \left(\frac{B_t}{T_c^*} \right) \nabla^* T^* \cdot \nabla^* T^* \right] \end{aligned} \quad (2.3)$$

$$\frac{\partial S^*}{\partial t^*} + \frac{1}{\epsilon} \mathbf{q}_d^* \cdot \nabla^* S^* = S_d \nabla^{*2} S^* + S_{ct} \nabla^{*2} T^* \quad (2.4)$$

$$\frac{\partial \psi^*}{\partial t^*} + \frac{1}{\epsilon} \mathbf{q}_d^* \cdot \nabla^* \psi^* = B_d \nabla^{*2} \psi^* + \frac{B_t}{T_r^*} \nabla^{*2} T^* \quad (2.5)$$

$$\left(\frac{\partial}{\partial t^*} + \frac{1}{\epsilon} (\mathbf{q}_d^* \cdot \nabla^*) \right) \mathbf{M}^* = (\mathbf{M}^* \cdot \nabla^*) \frac{1}{\epsilon} \mathbf{q}_d^* + \eta \nabla^{*2} \mathbf{M}^* \quad (2.6)$$

Here, equation (6) is Modified maxwell equation where $\nabla^* M^* = 0$, $\eta = \frac{1}{4\pi\mu_e\sigma'}$ and $\mathbf{q}_d^* = (u_{1d}^*, u_{2d}^*, u_{3d}^*)$ is Darcian velocity. For constant temperature and salt concentrations at the boundaries and zero nanoparticle flux, the boundary conditions are

$$\mathbf{q}_d^* = 0, T^* = T_l^*, S^* = S_l^*, B_d \frac{\partial \psi^*}{\partial z^*} + \frac{B_t}{T_r^*} \frac{\partial T^*}{\partial z^*} = 0 \quad \text{at} \quad z^* = 0 \quad (2.7)$$

$$\mathbf{q}_d^* = 0, T^* = T_r^*, S^* = S_r^*, B_d \frac{\partial \psi^*}{\partial z^*} + \frac{B_t}{T_r^*} \frac{\partial T^*}{\partial z^*} = 0 \quad \text{at} \quad z^* = a \quad (2.8)$$

Taking following non-dimensional parameters $(X, Y, Z) = \frac{(x^*, y^*, z^*)}{a}$, $t = \frac{t^* \alpha_m}{\sigma a^2}$, $(u_{1d}, u_{2d}, u_{3d}) = \frac{(u_{1d}^*, u_{2d}^*, u_{3d}^*) a}{\alpha_m}$, $p = \frac{p^* K}{\mu \alpha_m}$, $\psi = \frac{\psi^* - \psi_0^*}{\psi_0^*}$, $T = \frac{T^* - T_r^*}{T_l^* - T_r^*}$, $S = \frac{S^* - S_r^*}{S_l^* - S_r^*}$, $\lambda = \frac{\lambda^* \alpha_m}{a^2}$, $(\mathbf{M}_X, \mathbf{M}_Y, \mathbf{M}_Z) = \frac{(\mathbf{M}_X^*, \mathbf{M}_Y^*, \mathbf{M}_Z^*)}{M_0^*}$, where ψ_0^* is reference scale for volumetric fraction of nanoparticles, $\alpha_m \left(= \frac{k_m}{(\rho c)_F} \right)$ is the thermal diffusivity of the porous medium and $\sigma \left(= \frac{(\rho c)_M}{(\rho c)_F} \right)$ is the heat capacity ratio parameter. On replacing \mathbf{q}_d by \mathbf{q} , non-dimensional form of equations is given by

$$\nabla \cdot \mathbf{q} = 0 \quad (2.9)$$

$$\begin{aligned} \mathbf{q} - \mathbb{C} \nabla^2 \mathbf{q} &= \left(1 + \frac{\lambda}{\sigma} \frac{\partial}{\partial t} \right) \left[\left(-\nabla p - R_m \hat{e}_z - R_n \psi \hat{e}_z + R_a T \hat{e}_z + \frac{R_s}{Ln} S \hat{e}_z \right) \right. \\ &\quad \left. + \frac{P_1}{P_{1M}} QD_a (\nabla \times \mathbf{M}) \times \mathbf{M} \right] \end{aligned} \quad (2.10)$$

$$\frac{\partial T}{\partial t} + (\mathbf{q} \cdot \nabla) T = \nabla^2 T + \frac{N_b}{Le} \nabla \psi \cdot \nabla T + \frac{N_a N_b}{Le} \nabla T \cdot \nabla T \quad (2.11)$$

$$\frac{1}{\sigma} \frac{\partial S}{\partial t} + \frac{1}{\epsilon} \mathbf{q} \cdot \nabla S = \frac{1}{Ln} \nabla^2 S + N_{ct} \nabla^2 T \quad (2.12)$$

$$\frac{1}{\sigma} \frac{\partial \psi}{\partial t} + \frac{1}{\epsilon} (\mathbf{q} \cdot \nabla) \psi = \frac{1}{Le} \nabla^2 \psi + \frac{N_a}{Le} \nabla^2 T \quad (2.13)$$

$$\frac{1}{\sigma} \frac{\partial \mathbf{M}}{\partial t} + \frac{1}{\epsilon} (\mathbf{q} \cdot \nabla) \mathbf{M} = \frac{1}{\epsilon} (\mathbf{M} \cdot \nabla) \mathbf{q} + \frac{P_1}{P_{1M}} \nabla^2 \mathbf{M} \quad (2.14)$$

Here $R_a \left(= \frac{\rho g \beta K d (T_l^* - T_u^*)}{\mu \alpha_m} \right)$, $R_n \left(= \frac{\psi_0^* g K d (\rho_P - \rho)}{\mu \alpha_m} \right)$, $R_m \left(= \frac{\rho_P \psi_0^* + \rho (1 - \psi_0^*) g K d}{\mu \alpha_m} \right)$, $R_s \left(= \frac{\rho g \beta_c K d (S_l^* - S_u^*)}{\mu S_d} \right)$ are thermal, concentration, basic density and solutal Rayleigh Darcy number respectively, $\mathbb{C} \left(= \frac{\mu_{cs}}{\mu_l^2} \right)$ is couple-stress parameter, $P_1 \left(= \frac{\mu}{\rho \alpha_m} \right)$ and $P_{1m} \left(= \frac{\mu}{\rho n} \right)$ are Prandtl numbers, $Q \left(= \frac{\mu_e M_0^{*2} d^2}{4 \pi \mu \eta} \right)$ is Magnetic Chandrasekhar number, $D_a \left(= \frac{K}{l^2} \right)$ is Darcy number, $N_{cl} \left(= \frac{(\rho c)_P \epsilon Q_0^*}{(\rho c)_F} \right)$ are modified diffusivity ratio and modified particle density increment respectively, $Le \left(= \frac{\alpha_m}{B_d} \right)$ and $Ln = \frac{\alpha_m}{S_d}$ are Lewis numbers for nanofluid and salt respectively.

2.1. Basic State and Perturbed State Time independent basic state of nanofluid is described as

$$\mathbf{q} = 0, p = p_{bs}(Z), T = T_{bs}(Z), \psi = \psi_{bs}(Z), S = S_{bs}(Z), \mathbf{M} = \hat{e}_z \quad (2.15)$$

where the suffix “bs” refers to the basic flow. Following Chandrasekhar [3], the basic volume fraction and temperature of nanoparticles are given as $T_{bs} = 1 - Z$, $\psi_{bs} = \psi_0 + N_a Z$ and $S_{bs} = 1 - Z$. On the basic state, we superimpose perturbations in the form $\mathbf{q} = \mathbf{q}'$, $p = p_{bs} + p'$, $S = S_{bs} + S'$, $\psi = \psi_{bs} + \psi'$, $\mathbb{M} = \hat{e}_z + M'$

Linearised perturbation equations of Couple Stress nanofluid are obtained as

$$\begin{aligned} & \left(\frac{1}{\sigma} \frac{\partial}{\partial t} - \frac{P_1}{P_{1M}} \nabla^2 \right) \left[(\nabla^2 - \mathbb{C} \nabla^4) \mu'_{3d} - \left(1 + \frac{\lambda}{\sigma} \frac{\partial}{\partial t} \right) \left(R_a \nabla_H^2 T' - R_n \nabla_H^2 \psi' + \frac{R_s}{Ln} \nabla_H^2 S' \right) \right] \\ & = \left(1 + \frac{\lambda}{\sigma} \frac{\partial}{\partial t} \right) Q \frac{P_1}{P_{1M}} \frac{D_a}{\epsilon} \nabla^2 \frac{\partial^2 u'_{3d}}{\partial Z^2} \end{aligned} \quad (2.16)$$

$$\frac{\partial T'}{\partial t} - \mu'_{3d} = \nabla^2 T' - \frac{N_a N_b}{Le} \frac{\partial T'}{\partial Z} - \frac{N_b}{Le} \frac{\partial \psi'}{\partial Z} \quad (2.17)$$

$$\frac{1}{\sigma} \frac{\partial S'}{\partial t} - \frac{\mu'_{3d}}{\epsilon} = \frac{1}{Ln} \nabla^2 S' + N_{cl} \nabla^2 T' \quad (2.18)$$

$$\frac{1}{\sigma} \frac{\partial \psi'}{\partial t} + \frac{1}{\epsilon} N_a \mu'_{3d} = \frac{1}{Le} \nabla^2 \psi' + \frac{N_a}{Le} \nabla^2 T' \quad (2.19)$$

with the boundary conditions

$$\mu'_{3d} = 0, T' = 0, S' = 0, \frac{\partial \psi'}{\partial Z} + N_a \frac{\partial T'}{\partial Z} = 0 \quad \text{at} \quad Z = 0 \text{ and } Z = 1 \quad (2.20)$$

3. Linear Stability Analysis

Following the linear stability theory by Chandrasekhar [3], the perturbations are taken of the form

$$(\psi', T', \mu'_{3d}, S') = [\Phi(Z), \Theta(Z), \Omega(Z), \Psi(Z)]e^{st+iLX+iMY}, \quad (3.1)$$

where L and M are dimensionless wave numbers in X and Y directions respectively. On substituting the above values and employing Galerkin-type weighted residuals method with first approximation ($N = 1$), $\Omega = A_1 \sin \pi Z$, $B_1 = \sin \pi Z$, $\Phi = -N_a C_1 \sin \pi Z$, $\Psi = D_1 \sin \pi Z$. Substituting these values in equations (2.16)-(2.19) and taking the determinant of coefficient matrix as zero, the following Rayleigh number is obtained

$$R_a = \frac{\sigma}{\epsilon \alpha^2} \left[\frac{\begin{aligned} &R_s \alpha^2 (\lambda s + \sigma) (\sigma A \delta^2 + s) (\sigma \delta^2 + s Le) (\delta^2 + s) \{ \delta^2 (\epsilon N_{ct} - 1) - s \} \\ &- R_n N_a \alpha^2 (\lambda s + \sigma) (\delta^2 \sigma + s Ln) (A \delta^2 \sigma + s) \{ \delta^2 (\epsilon + Le) + s Le \} \\ &+ \epsilon (\delta^2 \sigma + s Ln) (\delta^2 \sigma + s Le) (\delta^2 + s) \{ A \sigma \delta^4 + B \pi^2 \delta^2 (\sigma + \lambda \delta) + s \delta^2 + s \mathbb{C} \delta^4 + A \mathbb{C} \sigma \delta^6 \} \end{aligned}}{(\sigma \delta^2 + s Ln) (\sigma \delta^2 + s Le) (\lambda s + \sigma) (A \delta^2 \sigma + s)} \right] \quad (3.2)$$

where $\delta^2 = \pi^2 + \alpha^2$. Taking $s=0$ in equation (3.2), the following Rayleigh number is obtained

$$R_a^{st} = \frac{\delta^4}{\alpha^2} - \left(1 + \frac{Le}{\epsilon} \right) R_n N_a + \frac{Q D_a \pi^2 \delta^2}{\epsilon \alpha^2} - \frac{R_s}{\epsilon} (1 - \epsilon N_{ct}) + \mathbb{C} \frac{\delta^6}{\alpha^2} \quad (3.3)$$

The above relation expresses the stationary Rayleigh number as a function of the parameters \mathbb{C} , Le , Q , N_a , R_n , ϵ , D_a , R_s , N_{ct} and dimensionless wave number α .

To obtain critical Rayleigh number putting $\frac{dR_a^{st}}{d\alpha} = 0$, critical wave number is given by equation

$$2\mathbb{C}(\alpha^2)^3 + (3\pi^2 \mathbb{C} + 1)(\alpha^2)^2 - \left(\pi^6 + \mathbb{C} \pi^4 + \frac{Q D_a}{\epsilon} \pi^4 \right) = 0 \quad (3.4)$$

which shows that critical wave number depends on Couple Stress parameter, Darcy number, Porosity and Magnetic field.

4. Results and Discussions

From equation (23), we have $\frac{\partial R_a^{st}}{\partial \mathbb{C}} = \frac{(\pi^2 + \alpha^2)^3}{\alpha^2}$ which is same as obtained for a couple stress regular fluid by Bishnoi et al. [10]. This shows that the effect of couple stress parameter for a nanofluid layer is to stabilize the stationary convection in same way as for a regular fluid. If $R_n = 0$, $Q = 0$, $R_s = 0$, then $R_a^{st} = \frac{\delta^4}{\alpha^2} + \mathbb{C} \frac{\delta^6}{\alpha^2} = \frac{(\pi^2 + \alpha^2)^2}{\alpha^2} (1 + \mathbb{C} \delta^2)$ which is same as obtained by Shivakumara [11]. The stationary convection curves for Rayleigh number R_a versus the wave number α are shown in Fig. 1(a)-(i) by assigning fixed values, $\mathbb{C} = 5$, $N_a = 4$, $D_a = 0.2$, $Le = 10$, $R_n = 4$, $\epsilon = 0.4$, $Q = 800$, $R_s = 5$, $N_{ct} = 0.1$ with variations in one of these parameters. For oscillatory convection, R_a vs α curves are shown in Fig. 2(a)-(l) for fixed value of R_n , Le , Q , D_a , ϵ , N_A , σ , λ , \mathbb{C} , Ln , R_s and N_{ct} with variations in one of these parameters.

No critical Rayleigh number is obtained for oscillatory convection. The comparison of results obtained has been done with the existing relevant studies. The outcomes of the present analysis are summarised as follows: R_a^{st} has been observed to be function of parameters \mathbb{C} , D_a , ϵ , Le , R_n , N_a , Q , N_{ct} , R_s whereas R_a^{osc} is function of Ln , σ and λ in addition to above parameters.

The couple stress parameter has been found to stabilize the stationary convection as observed by Chand et al. [12] too while studying thermal instability in a layer of couple stress nanofluid in absence of magnetic field. But opposite to non-occurrence of oscillatory convection there, couple stress is found to have destabilizing effect on oscillatory convection here. The effect of Lewis number Le is to decrease Rayleigh number. An increase in porosity decreases R_a^{osc} but dual effect on stationary Rayleigh number.

A positive Soret coefficient N_{ct} has stabilizing effect on convection as obtained by Gaikwad et al. [13] for a regular fluid as well as obtained by Singh et al. [14] for a nanofluid but here in presence of magnetic field and couple stress parameter the effect is found to destabilizing the oscillatory convection as increase in N_{ct} decreases R_a^{osc} . The influence of magnetic field is to stabilise the Soret induced double diffusive convection as was found by Yadav [15] in nanofluid convection induced by internal heating. In this convection under magnetic field, Darcy number also comes into play and has been observed to provide stabilizing effect on stationary and oscillatory modes. An increase in Solutal Rayleigh Darcy number R_s was observed to cause increase in R_a^{st} and R_a^{osc} by Singh et al. [14] but the presence of magnetic field here causes the effect of R_s to be stabilize the oscillatory convection. The increase in parameters Ln , σ decreases the oscillatory Rayleigh number. This behaviour is same as observed in absence of magnetic field and couple stress parameter by Singh et al. [14]. Parameters R_n and N_a destabilizes the stationary mode but are found to stabilize the oscillatory convection.

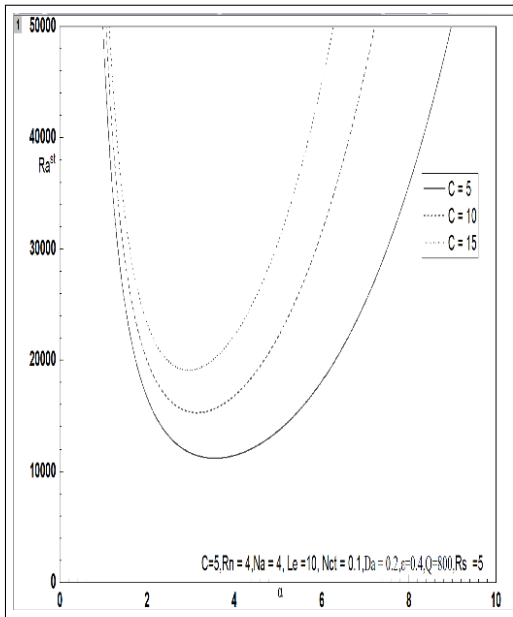


Fig. 1(a)

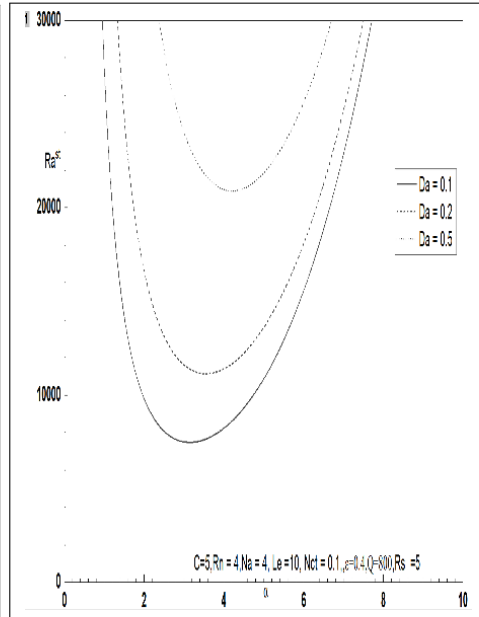


Fig. 1(b)

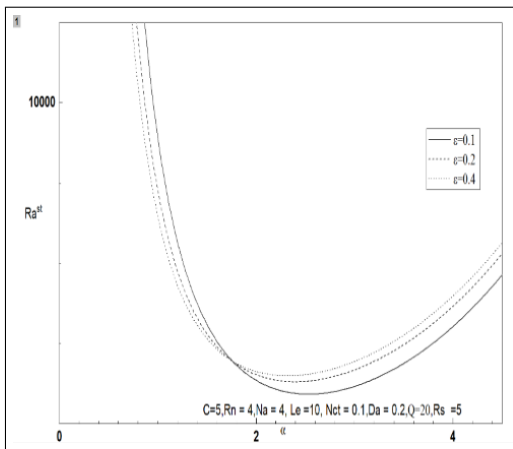


Fig. 1(c)

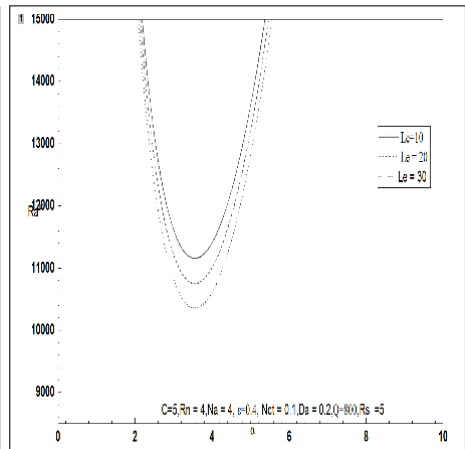


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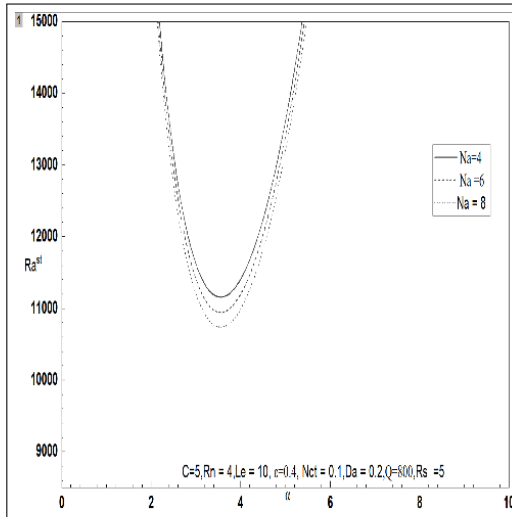


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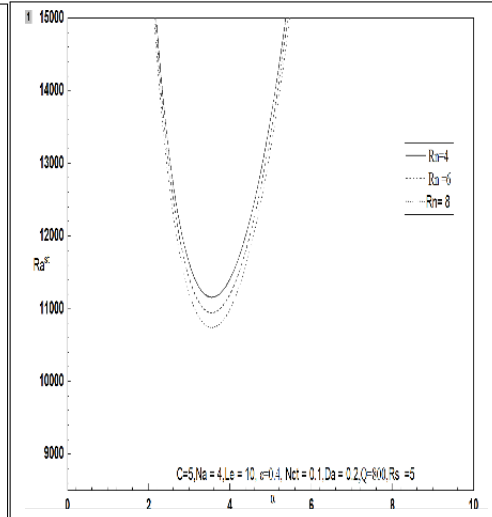


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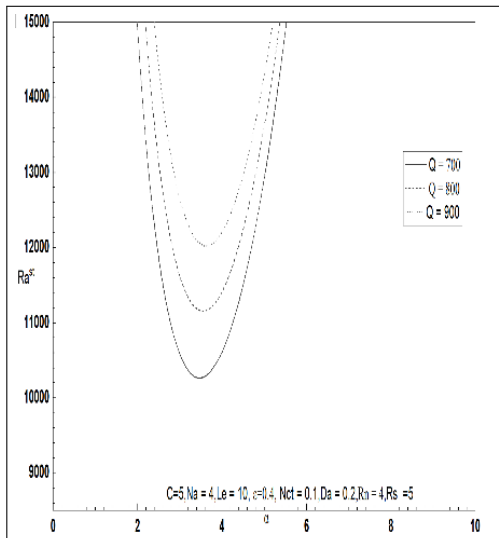


Fig. 1(g)

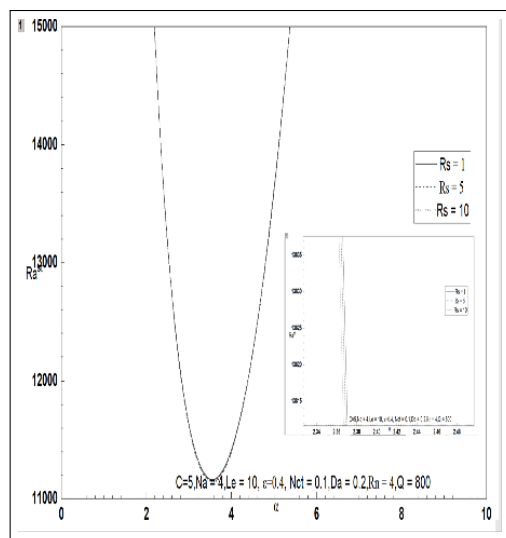


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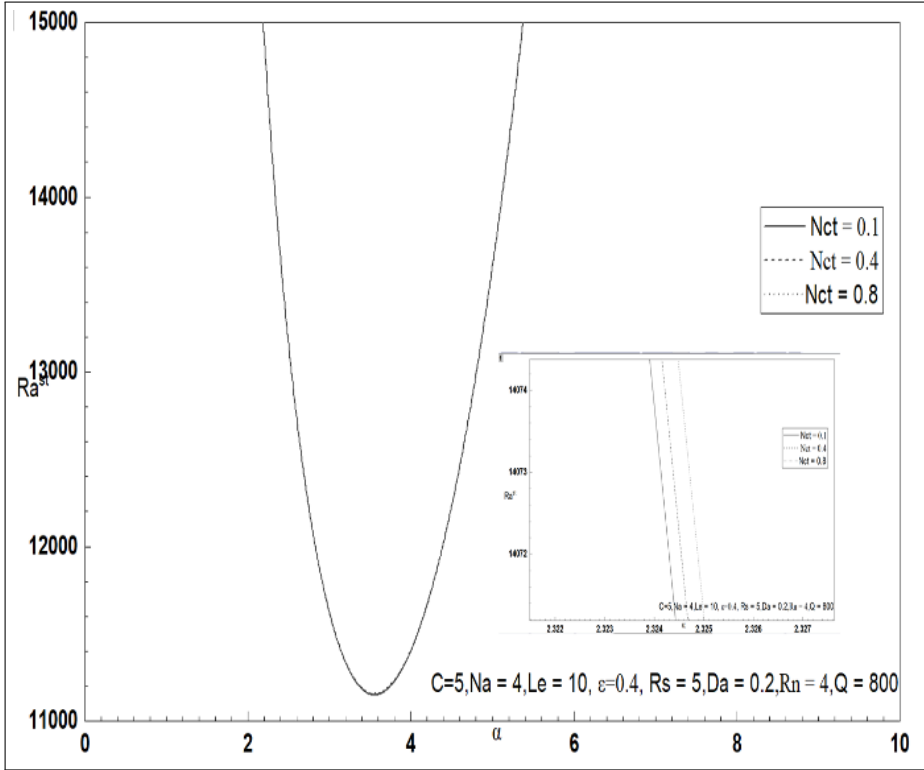


Fig. 1(i)

Fig. 1: Linear stationary convection with wave number α for different values of
 (a) C , (b) Da , (c) ϵ , (d) Le , (e) Na , (f) Rn , (g) Q , (h) Rs , (i) N_{ct}

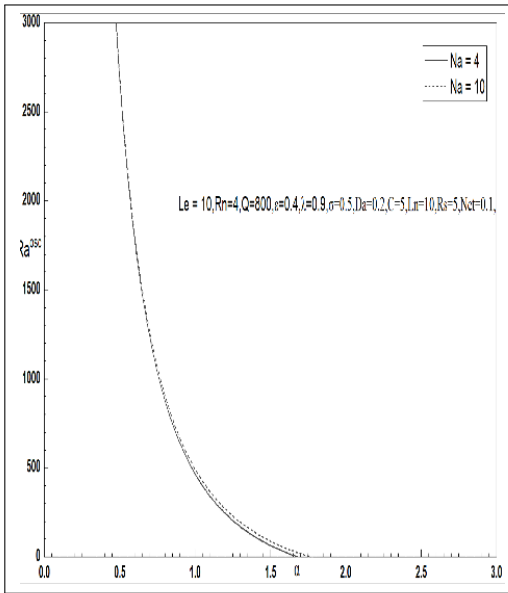


Fig. 2(a)

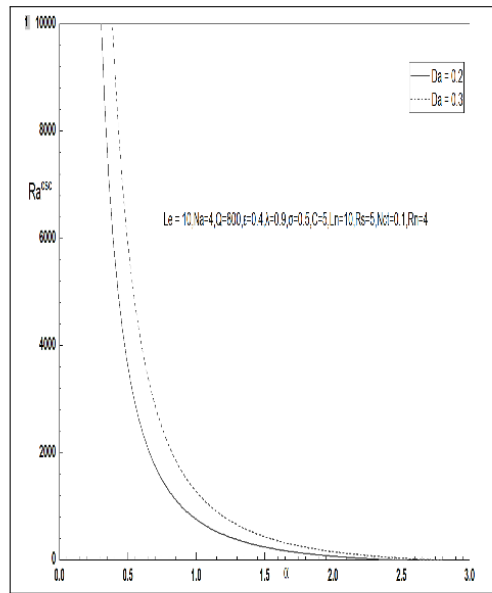


Fig. 2(b)

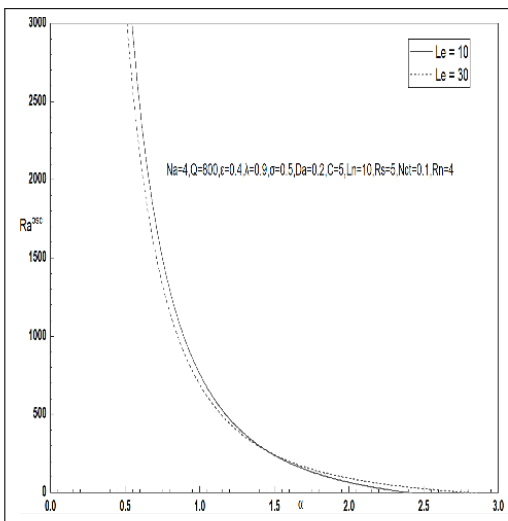


Fig. 2(c)

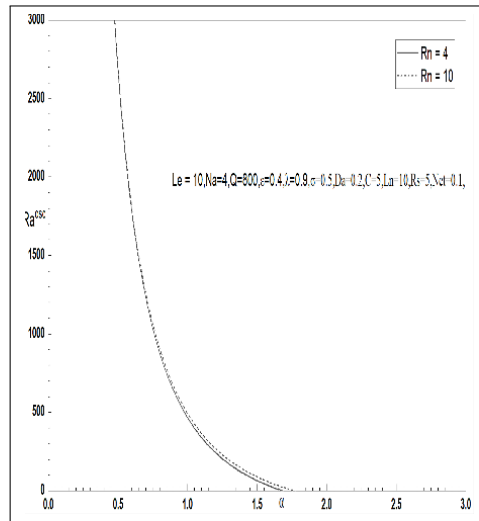


Fig. 2(d)

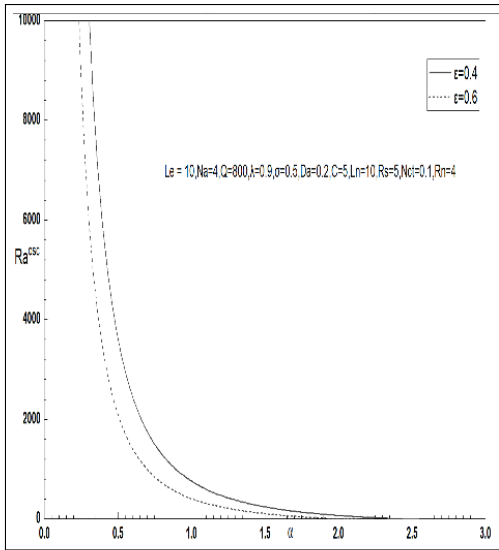


Fig. 2(e)

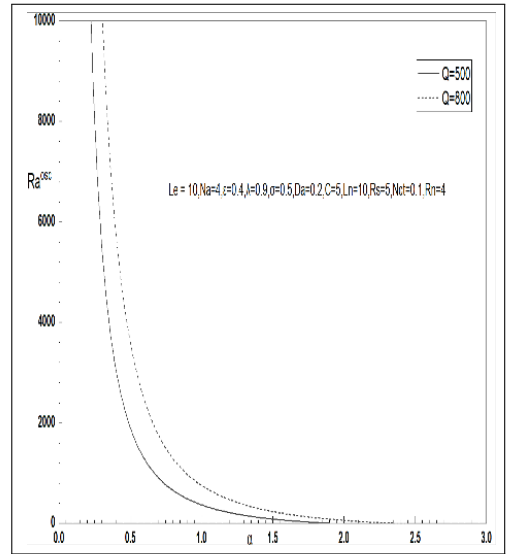


Fig. 2(f)

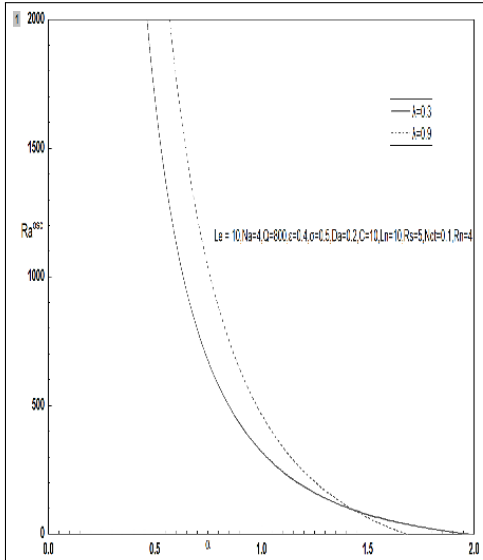


Fig. 2(g)

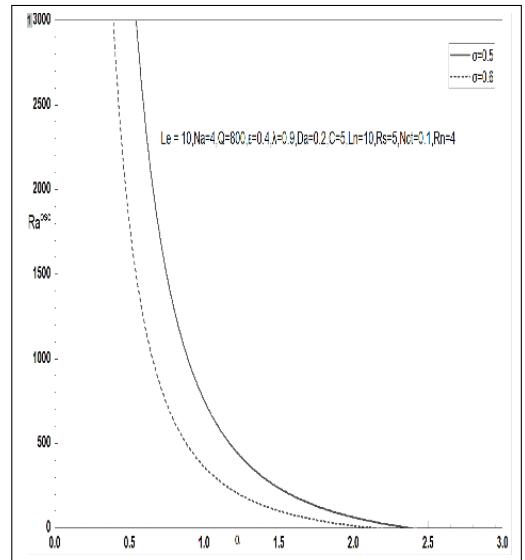


Fig. 2(h)

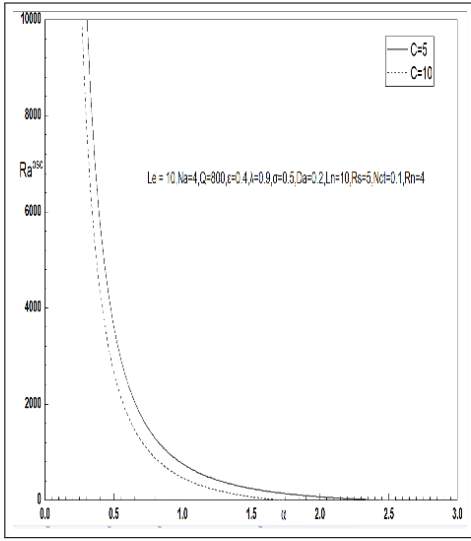


Fig. 2(i)

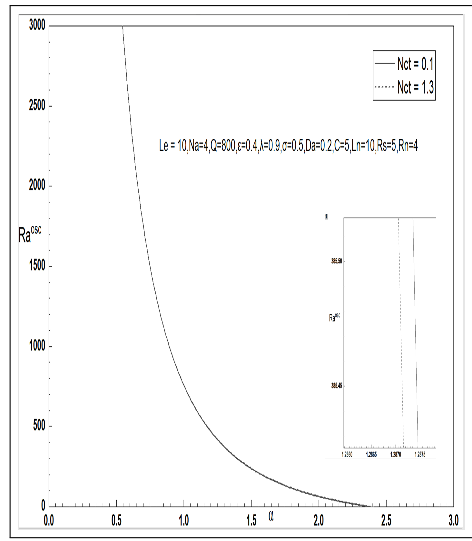


Fig. 2(j)

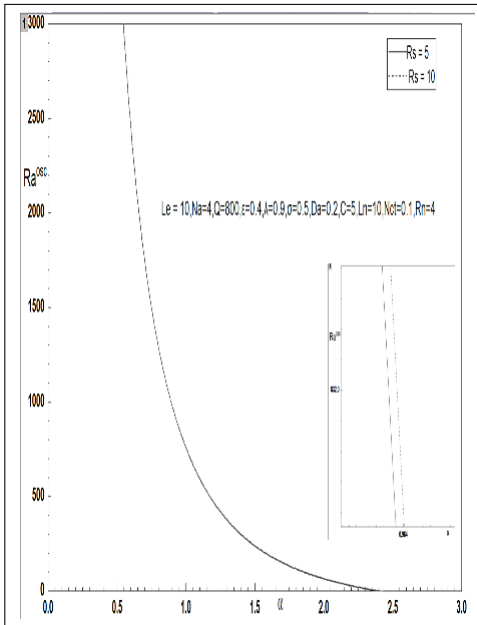


Fig. 2(k)

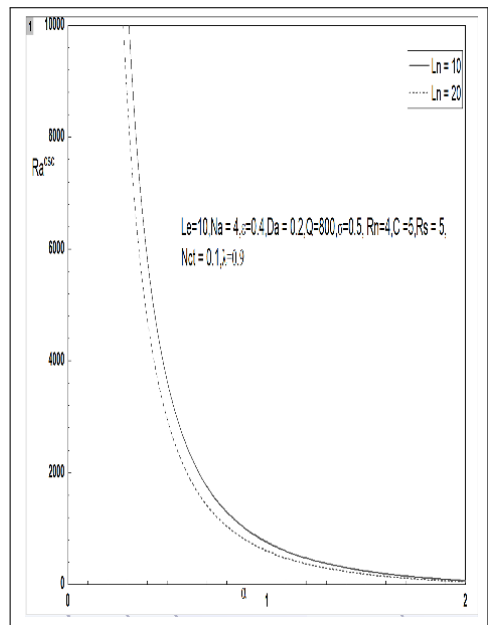


Fig. 2(l)

Fig. 2: Linear oscillatory convection with wave number α for different values of (a) N_a , (b) D_a , (c) Le , (d) R_n , (e) ϵ , (f) Q , (g) λ , (h) σ , (i) \mathbb{C} , (j) N_{ct} , (k) R_s , (l) Ln

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