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BASIC SOFT INEQUALITY ON A SOFT MATRIC SPACES AND ITS APPLICATIONS

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Abstract

Soft set theory, proposed by Molodtsov in 1999 [10] and provides an effective mathematical tool for dealing with uncertainties. In this paper, first we introduced basics of soft inequality in soft metric space with respect to the soft points. Then we discuss the basic prepositions of soft inequality on soft metric space such as soft function with soft points is monotonically decreasing and also monotonically increasing. Moreover, also we prove some Theorems based on soft inequality of soft sets and investigate its properties. Also we have given applications related to the soft inequality and medical diagnosis. The main purpose of this paper is to extend the theoretical aspect of soft inequality on soft metric space.

Keywords and phrases: LaTeX, soft metric space, soft distance function, soft points, soft function, soft inequality, soft monotonic increasing function and soft monotonic decreasing function.

1. Introduction

There are many difficult problems are available in the field of medical science, social science, engineering, economics, environment, etc. To solve that type of problems we cannot successfully use classical methods because there are a different types of uncertainties are present in these problems. The reason of that type of uncertainty molodtsov [10] introduced parameterized tools in 1990 and introduced the concept of soft set theory as a mathematical tool for dealing with uncertainties and these tools are free from the above difficulties. Soft set theory has a rich potential for applications in several directions, few of which had been shown by Molodtsov in his pioneer work. Soft set theory offers a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. The study of the theory of soft sets initiated by Molodtsov. [8] defined equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union, intersection are defined. De-Morgan's laws and a number of results are verified in soft set theory. [9] introduced the idea of soft mappings and some of their properties. They also introduced Images, inverse images of crisp sets, soft sets under soft mappings. Mapping is a fundamental mathematical concept which is used in many fundamental areas of science and mathematics and has numerous applications. One can also investigate the theory of fuzzy soft and intuitionistic fuzzy soft mappings as well as application of soft mapping in medical diagnosis. [22] introduced soft topological spaces which are defined over an initial universe with a fixed set of parameters. The notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms are introduced and their basic properties are investigated. They also show that a soft topological space gives a parameterized family of topological spaces. Furthermore, with the help of an example it is established that the converse does not hold. The soft subspaces of a soft topological space are defined and inherent concepts as well as the characterization of soft open and soft closed sets in soft subspaces are investigated. Finally, soft T1-spaces and notions of soft normal and soft regular spaces are discussed in detail. A sufficient condition for a soft topological space to be a soft T1-space is also presented in their work. According to[4] Matrices play an important role in the broad area of science and engineering. However, the classical matrix theory sometimes fails to solve the problems involving uncertainties, occurring in an imprecise environment. In initiated a matrix representation of a soft set and successfully applied the proposed notion of soft matrix in certain decision making problems. Also the soft max - min decision making method was set up in it. [23] introduced soft generalized topology on a soft set. As well as the definitions of subspace soft generalized topology and soft continuity of soft functions and Some basic concepts in soft generalized topological spaces are also defined and studied their properties. [11] introduced soft set theory and investigate various properties of soft metric spaces including soft dense, nowhere soft dense set, soft first category, soft second category, soft Baire space and soft isometric spaces. They also establish Baire's category theorem and completion theorem of a soft metric space. [13] introduced the distance between two soft points encountered in many practical applications such as soft distance that cannot be defined in soft topological spaces. They generated the soft metric spaces whose structure is represented by soft distance function shown by $\tilde{d}$. The description of $\tilde{d}$ provides an exact method to analyse the distance between two soft points. By using this method, they studied new classes of soft mappings. They also presented different definitions of d' in the sense that they provided soft metric spaces that were not described before. [14] introduced soft point's soft matrix form which were not described before is defined for each set of parameters. The matrix representation of soft points is useful for storing all soft points that can be obtained in all different parameters. The proposed soft matrix provides every soft point that changes with each parameter that takes place in a soft set is proved and showed that it enables detailed examination in application of soft set theory. [2] contributed for investigating on soft S-metric space which is based on soft point of soft sets and gave some of their properties. Moreover, they also introduced the concepts of soft continuous and soft sequentially continuous mapping and examine the connection between them.[12] introduced the basic elements of the theory, including the notion of a soft set, the operations on soft sets, and their semantic interpretations. They describes various T-generalizations and modifications of soft set theory, such as N-soft sets, fuzzy soft sets, and bipolar soft sets, highlighting their specific characteristics. Furthermore, they also given the fundamentals of various extensions of mathematical structures from the perspective of soft set theory. Particularly they define basic results of soft topology and other algebraic structures such as soft algebras and r-algebras. There survey concludes with a discussion on the challenges and future directions in soft set theory, emphasizing the need for further research to enhance its theoretical foundations and broaden its practical applications. Overall, the survey of soft set theory serves as a valuable resource for practitioners, researchers, and students interested in understanding and utilizing this flexible mathematical framework for tackling uncertainty in decision-making processes. [20] introduced the boundaries of what is possible with multi-criteria decision-making frameworks, the role of octahedron sets is poised to become increasingly central. there progression not only promises to enhance the efficacy and efficiency of decision processes but also paves the way for pioneering research and applications that could redefine problem solving in numerous domains.

2. Preliminaries:

In this section, we present the notion of *soft set*, which was introduced by Molodtsov in 1999 to deal with uncertainty in a non-parametric manner. A soft set is a collection of approximate descriptions of an object. A soft set is a parameterized family of sets — intuitively, this is "soft" because the boundary of the set depends on the parameters.

Formally, a soft set over a universal set X and a set of parameters E is a pair (f, A), where $A \subseteq E$, and f is a function from A to the power set of X, i.e., $f : A \to \mathcal{P}(X)$. For each $e \in A$, the set $f(e)$ is called the *value set* of e in (f, A).

Definition 2.1. [10] Let U be an initial universal set and E a set of parameters or attributes with respect to U. Let $\mathcal{P}(U)$ denote the power set of U, and let $A \subseteq E$. A pair (F, A) is called a *soft set* over U, where F is a mapping given by:

$$F : A \to \mathcal{P}(U).$$

In other words, a soft set (F, A) over U is a parameterized family of subsets of the universe U.

For $e \in A$, $F(e)$ may be considered as the set of ϵ-approximate elements of the soft set (F, A). Clearly, a soft set is not a classical set.

For illustration, Molodtsov considered several examples, one of which we present below.

Example 2.2. *Suppose the following:*

Let U be the set of hotels under consideration, and E be the set of parameters. Each parameter is a word or a sentence.

E = {expensive, beautiful, wooden, cheap, in the green surroundings, modern, in good repair, in bad repair }.

In this case, to define a soft set means to point out expensive hotels, beautiful hotels, and so on. The soft set (F, E) *describes the attractiveness of the hotels which Mr. X (say) is going to buy.*

We consider the same example in more detail for our next discussion.

Suppose the universe of hotels is given by:

$$U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$$

and let us consider a subset of parameters:

$$E = \{e_1, e_2, e_3, e_4, e_5\}$$

where:

- e_1 *stands for the parameter expensive,*
- e_2 *stands for the parameter beautiful,*
- e_3 *stands for the parameter wooden,*
- e_4 *stands for the parameter cheap,*
- e_5 *stands for the parameter in the green surroundings.*

The soft set F is then defined as:

$$\begin{aligned} F(e_1) &= \{h_2, h_4\}, \\ F(e_2) &= \{h_1, h_3\}, \\ F(e_3) &= \{h_3, h_4, h_5\}, \\ F(e_4) &= \{h_1, h_3, h_5\}, \\ F(e_5) &= \{h_1\}. \end{aligned}$$

The soft set (F, E) is a parameterized family $\{F(e_i),\ i = 1, 2, 3, 8\}$ of subsets of the set U, and it gives us a collection of approximate descriptions of an object.

Consider the mapping F, which can be interpreted as *"hotels(.)"*, where the dot "**.**" is to be filled by a parameter $e \in E$. Therefore, $F(e_1)$ means *"hotels (expensive)"*, whose functional value is the set $\{h_2, h_4\}$.

Thus, we can view the soft set (F, E) as a collection of approximations as follows:

$$(F, E) = \begin{cases} \text{expensive hotels} = \{h_2, h_4\}, \\ \text{beautiful hotels} = \{h_1, h_3\}, \\ \text{wooden hotels} = \{h_3, h_4, h_5\}, \\ \text{cheap hotels} = \{h_1, h_3, h_5\}, \\ \text{in the green surroundings} = \{h_1\}. \end{cases}$$

Each approximation in the soft set has two parts:

(i) A **predicate** p,

(ii) An **approximate value-set** u (or simply called the value-set).

Example: For the approximation *expensive hotels* $= \{h_2, h_4\}$:

(i) The predicate name is *expensive hotels*,

(ii) The approximate value-set is $\{h_2, h_4\}$.

Definition 2.3 ([8]). A soft set (F, A) over U is said to be a *null soft set*, denoted by $\emptyset$, if for all $e \in A$, $F(e) = \emptyset$.

Definition 2.4 ([8]). A soft set (F, A) over U is said to be an *absolute soft set*, denoted by $\tilde{A}$, if $\forall e \in A,\ F(e) = U$. Clearly, $\tilde{A}^C = \tilde{A}$.

Definition 2.5 ([9]). Let A, B be two non-empty sets and E' be a parameter set. Then the mapping

$$F : E' \to \mathcal{P}(B^A)$$

is called a *soft mapping* from A to B under E', where B^A is the set of all mappings from A to B. In fact, a soft mapping F from A to B under E' is a soft set over B^A.

Definition 2.6 ([9]). Let A, B, C be non-empty sets and E', E_1, E_2 be parameter sets. Then the soft mapping

$$F : E' \to \mathcal{P}(B^A), \quad \text{defined by } F(e) = \{i_A\} \text{ for all } e \in E',$$

where $i_A : A \to A$ is the identity mapping in A, is called the *identity soft mapping* on A under E'.

Definition 2.7 ([9]). A soft mapping $F : E' \to \mathcal{P}(B^A)$ is said to be a *constant soft mapping* under E' if $\forall e \in E'$, $F(e)$ is a collection of constant mappings from A to B.

Definition 2.8 ([9]). Let $F_1 : E_2 \to \mathcal{P}(B^A)$ and $F_2 : E_2 \to \mathcal{P}(B^A)$ be two soft mappings over (U, E). They are said to be *equal*, denoted by $F_1 = F_2$, if:
(i) $E_1 = E_2$, and
(ii) $F_1(e) = F_2(e)$ for all $e \in E_1 = E_2$.

Definition 2.9. [9] A soft mapping $F : E' \to \mathcal{P}(B^A)$ is said to be:
- *Weakly injective* if $\forall e, f \in E', e \neq f \Rightarrow F(e) \neq F(f)$.
- *Strongly injective* if $\forall e, f \in E', e \neq f \Rightarrow F(e) \cap F(f) = \varnothing$.

Definition 2.10. [9] A soft mapping $F : E' \to \mathcal{P}(B^A)$ is said to be:
- *Weakly surjective* if for any $f \in B^A$, there exists $e \in E'$ such that $f \in F(e)$.
- *Strongly surjective* if for all $f \in B^A$, $f \in F(e)$ for every $e \in E'$.

Definition 2.11. [9] A soft mapping $F : E \to \mathcal{P}(B^A)$ is said to be:
- *Naturally injective* if for all $e \in E$, $F(e)$ is a collection of injective mappings from A to B.
- *Naturally surjective* if for all $e \in E$, $F(e)$ is a collection of surjective mappings from A to B.

If F is both naturally injective and naturally surjective, then F is called *naturally bijective*.

Definition 2.12. [13] Let $\emptyset \neq X \subseteq E$, $f \in S_X(U)$ and $f : X \to \mathcal{P}(U)$ be a one-to-one function. Let $f_i, f_j, f_s \in S_X(U)$ and $(e_{i_f})_i$, $(e_{j_f})_j$, $(e_s)_s \in f$. A mapping

$$\tilde{d} : SP(f) \times SP(f) \to \tilde{\mathbb{R}}(E)$$

is said to be a *soft metric* on the soft set f if $\tilde{d}$ satisfies the following conditions:

(i) $\tilde{d}((e_{i_f})_i, (e_{j_f})_j) \geq 0$,

(ii) $\tilde{d}((e_{i_f})_i, (e_{j_f})_j) = 0 \Leftrightarrow (e_{i_f})_i = (e_{j_f})_j$,

(iii) $\tilde{d}((e_{i_f})_i, (e_{j_f})_j) = \tilde{d}((e_{j_f})_j, (e_{i_f})_i)$ (Symmetric Axiom),

(iv) $\tilde{d}((e_{i_f})_i, (e_{j_f})_j) \leq \tilde{d}((e_{i_f})_i, (e_s)_s) + \tilde{d}((e_{s_f})_s, (e_{j_f})_j)$ (Triangular Inequality Axiom).

If $\tilde{d}$ is a soft metric on the soft set f, then $(f, \tilde{d})$ is called a *soft metric space*.

Definition 2.13. [13] The soft set f is called a *soft point* in S if there exists a parameter $e_i \in E$ such that $f(e_i) \neq \emptyset$ and $f(e_k) = \emptyset$ for all $e_k \in E \setminus \{e_i\}$. This soft point is denoted by $(e_{i_f})_j$ for all $i, j, k \in \mathbb{N}^+$ with $i \neq j \neq k$. The set of all soft points of f is denoted by $SP(f)$.

The axiom (i) of soft metric spaces (non-negativity) is a consequence of the other three axioms. Let $(f, \tilde{d})$ be a soft metric space. Then, for all $(e_{i_f})_i, (e_{j_f})_j, (e_s)_s \in \tilde{f}$:

$$0 = \tilde{d}((e_{i_f})_i, (e_{i_f})_i) \leq \tilde{d}((e_{i_f})_i, (e_{j_f})_j) + \tilde{d}((e_{j_f})_j, (e_{i_f})_i) = 2\tilde{d}((e_{i_f})_i, (e_{j_f})_j)$$

This implies:

$$\tilde{d}((e_{i_f})_i, (e_{j_f})_j) \geq 0$$

Thus, it is not necessary to prove axiom (i) when verifying that $\tilde{d}$ is a soft metric.

Definition 2.14. [13] Let $(e_{i_f})_i$ and $(e_{j_f})_j$ be soft points in a soft metric space. The value of $\tilde{d}((e_{i_f})_i, (e_{j_f})_j)$ is called the *soft distance* between the soft points $(e_{i_f})_i$ and $(e_{j_f})_j$.

Proposition 2.15. *[13] Let $(e_{i_f})_i, (e_{j_f})_j \in \tilde{f}$ for all $i, j \in \mathbb{N}$ and let $\tilde{r} \in \tilde{\mathbb{R}}(E)$. Then the function*

$$\tilde{d}((e_{i_f})_i, (e_{j_f})_j) = \begin{cases} 0, & \text{if } a_{ki} = a_{kj} \\ \tilde{r}, & \text{if } \tilde{r} \geq 0 \text{ and } a_{ki} \neq a_{kj} \end{cases}$$

defines a soft function on soft sets, and this function $\tilde{d}$ is a soft metric on f.

Proposition 2.16. *[13] Let $(e_{i_f})_i, (e_{j_f})_j \in \tilde{f}$, $\forall k \in \mathbb{N}$ and $\tilde{r}, \tilde{s} \in \tilde{\mathbb{R}}(E)$. Then the soft metric is defined by:*

$$\tilde{d}((e_{i_f})_i, (e_{j_f})_j) = |\tilde{r} - \tilde{s}|$$

Proposition 2.17 ([13]). *Let $(f, \tilde{d})$ be a soft metric space. For all $(e_{i_f})_i, (e_{j_f})_j \in \tilde{f}$ and $\tilde{r} \in \tilde{\mathbb{R}}(E)$, the soft function*

$$\tilde{d}_s((e_{i_f})_i, (e_{j_f})_j) = \min\left\{\tilde{d}((e_{i_f})_i, (e_{j_f})_j), \tilde{r}\right\}$$

is also a soft metric on f.

Proposition 2.18. *[15] Any soft intersection of soft open spheres need not be a soft open set, and any soft union of soft closed spheres need not be a soft closed set.*

Definition 2.19. [14] Let us denote the collection of all soft points of f by $SP(f)$, the set of all soft real sets by $\mathbb{R}(E)$, the set of all soft real numbers by $\tilde{\mathbb{R}}(E)$ and the set of all non-negative soft real numbers by $\tilde{\mathbb{R}}^*(E)$. If a soft real set is a singleton soft set, it is called a soft real number and denoted by $\tilde{r}$, $\tilde{s}$, etc.

Definition 2.20. [14] If $f(e) = \emptyset$ for all $e \in E$, then f is called a null soft point, denoted by e_φ.

Definition 2.21. [14] If $f(e) = U$ for all $e \in E$, then f is called a universal soft point, denoted by $e_{\tilde{E}}$. If there is only one parameter $e \in E$ in f, then f is denoted by e_{f_i}. If $f(e) = \{u\}$ for that single parameter, then f is denoted by e_f.

Definition 2.22. [14] The soft points $(e_{i_f})_j$ and $(e_{k_f})_s$ are said to be equal if $f(e_i) = f(e_k)$.

Definition 2.23. [11] Let $(\tilde{X}, d, E)$ and $(\tilde{Y}, d^*, E')$ be soft metric spaces and $\phi_{fu} : (\tilde{X}, d, E) \to (\tilde{Y}, d^*, E')$ be a soft mapping, where $f : X \to Y$ and $u : E \to E'$. Then ϕ_{fu} is called a soft isometry or soft isometric mapping if and only if

$$d^*(\phi_{fu}(T^a_w), \phi_{fu}(T^b_\mu)) = d(T^a_w, T^b_\mu)$$

for all $T^a_w, T^b_\mu \in \tilde{X}$. If ϕ_{fu} is bijective, then $(\tilde{X}, d, E)$ and $(\tilde{Y}, d^*, E')$ are called soft isometric spaces.

Definition 2.24. [17] An $F_A = [F^-_A, F^+_A]$ is called an interval-valued soft set (IVSS) over X, if $F_A : A \to IVS(X)$ is a mapping such that $F_A(e) = \tilde{\varphi}$ for each $e \notin A$, and $F^-_A, F^+_A \in SS(X)$ such that $F^-_A(e) \subset F^+_A(e)$ for each $e \in A$. It is clear that if $F_A \in SS(X)$, then $[F_A, F_A] \in IVSS(X)$. Thus, IVSS is a generalization of soft sets. Moreover, if $F_A \in IVSS(X)$, then X_{F_A} is an interval-valued fuzzy soft set.

Definition 2.25. [21] A soft topological space (X, τ, E) is said to be **soft countably compact**, if every countable soft open cover of X has a finite subcover. Clearly, every soft compact space is soft countably compact.

Definition 2.26. [21] A soft topological space (X, τ, E) is said to have the **Bolzano-Weierstrass property** if every infinite soft set over X has a soft limit point.

3. Main Result

In mathematics, if we compare two quantities as less than or greater than, and also compare one or more solutions, this concept is called **inequality**. We solve inequalities to find the solution of a given variable. It is about finding the relative order of a set. Inequalities might have many solutions, but usually, only the solutions as real numbers are the ones we are looking for. The proper way to read an inequality is from left to right, just like other equations, but the only difference is that inequalities have different rules for every equation.

Here, we introduce basic **soft inequalities** on soft metric spaces and their applications. The study of soft metric spaces is no exception. In fact, the definition of a soft metric space involves a soft inequality, which is a generalization of the familiar triangle inequality with respect to the soft points, satisfied by the soft distance function in $\tilde{R}$.

$$|(e_{(x_f)})_x - (e_{(y_f)})_y| \leq |(e_{(x_f)})_x - (e_{(z_f)})_z| + |(e_{(z_f)})_z - (e_{(y_f)})_y| \quad \forall (e_{(x_f)})_x, (e_{(y_f)})_y, (e_{(z_f)})_z \in \tilde{R}.$$

For n soft points, a basic soft inequality in soft metric spaces, which obviously follows from the triangle inequality, is the generalized triangle inequality, or the polygonal inequality.

If $\tilde{d}$ is a soft metric on the soft set f, then f is called a soft metric space and denoted by $(f, \tilde{d})$.

For any soft points $(e_{(1_f)})_1, (e_{(2_f)})_2, (e_{(3_f)})_3, \ldots, (e_{(n-1_f)})_{n-1}, (e_{(n_f)})_n$ $(n \geq 3)$ in a soft metric space $(f, \tilde{d})$, we have the inequality

$$\tilde{d}((e_{(1_f)})_1, (e_{(n_f)})_n) \leq \tilde{d}((e_{(1_f)})_1, (e_{(2_f)})_2) + \cdots + \tilde{d}((e_{(n-1_f)})_{n-1}, (e_{(n_f)})_n).$$

In this section, we establish some soft inequalities that will be required to confirm some of the examples.

Proposition 3.1. *The soft function* $f((e_{(a_f)})_a) = \frac{(e_{(a_f)})_a}{1+(e_{(a_f)})_a}$, *where* $(e_{(a_f)})_a \geq 0$, *with soft points is monotonically increasing.*

Proof. Suppose that $(e_{(b_f)})_b > (e_{(a_f)})_a \geq 0$. Then,

$$\frac{1}{1 + (e_{(b_f)})_b} < \frac{1}{1 + (e_{(a_f)})_a}.$$

Now,

$$1 - \frac{1}{1 + (e_{(b_f)})_b} > 1 - \frac{1}{1 + (e_{(a_f)})_a},$$

such that

$$\frac{(e_{(b_f)})_b}{1 + (e_{(b_f)})_b} > \frac{(e_{(a_f)})_a}{1 + (e_{(a_f)})_a}.$$

□

Proposition 3.2. *The soft function* $f((e_{(a_f)})_a) = \frac{(e_{(a_f)})_a}{1-(e_{(a_f)})_a}$, *where* $(e_{(a_f)})_a \geq 0$, *with soft points is monotonically decreasing.*

PROOF. Suppose that $(e_{(b_f)})_b > (e_{(a_f)})_a \geq 0$. Then,

$$\frac{1}{1-(e_{(b_f)})_b} < \frac{1}{1-(e_{(a_f)})_a}.$$

Now,

$$1-\frac{1}{1-(e_{(b_f)})_b} > 1-\frac{1}{1-(e_{(a_f)})_a},$$

such that

$$\frac{-(e_{(b_f)})_b}{1-(e_{(b_f)})_b} > \frac{-(e_{(a_f)})_a}{1-(e_{(a_f)})_a}.$$

$$\frac{(e_{(b_f)})_b}{1-(e_{(b_f)})_b} < \frac{(e_{(a_f)})_a}{1-(e_{(a_f)})_a}.$$

□

THEOREM 3.3. *For any two soft points $(e_{(a_f)})_a$ and $(e_{(b_f)})_b$, the following soft inequality holds:*

$$\frac{|(e_{(a_f)})_a + (e_{(b_f)})_b|}{1+|(e_{(a_f)})_a + (e_{(b_f)})_b|} \leq \frac{|(e_{(a_f)})_a|}{1+|(e_{(a_f)})_a|} + \frac{|(e_{(b_f)})_b|}{1+|(e_{(b_f)})_b|}.$$

PROOF. Let the two soft points be $(e_{(a_f)})_a$ and $(e_{(b_f)})_b$, and assume they have the same sign. Without loss of generality, we may assume that $(e_{(a_f)})_a \geq 0$ and $(e_{(b_f)})_b \geq 0$.

Now,

$$\frac{|(e_{(a_f)})_a + (e_{(b_f)})_b|}{1+|(e_{(a_f)})_a + (e_{(b_f)})_b|} = \frac{(e_{(a_f)})_a + (e_{(b_f)})_b}{1+(e_{(a_f)})_a + (e_{(b_f)})_b}$$

$$= \frac{(e_{(a_f)})_a}{1+(e_{(a_f)})_a + (e_{(b_f)})_b} + \frac{(e_{(b_f)})_b}{1+(e_{(a_f)})_a + (e_{(b_f)})_b}.$$

Thus, we have

$$\leq \frac{(e_{(a_f)})_a}{1+(e_{(a_f)})_a} + \frac{(e_{(b_f)})_b}{1+(e_{(b_f)})_b} = \frac{|(e_{(a_f)})_a|}{1+|(e_{(a_f)})_a|} + \frac{|(e_{(b_f)})_b|}{1+|(e_{(b_f)})_b|}.$$

Suppose the two soft points $(e_{(a_f)})_a$ and $(e_{(b_f)})_b$ have different signs. Here, we may assume that $|(e_{(a_f)})_a| \geq |(e_{(b_f)})_b|$. Then, we have

$$|(e_{(a_f)})_a + (e_{(b_f)})_b| \leq |(e_{(a_f)})_a|.$$

From the above results, it follows that

$$\frac{|(e_{(a_f)})_a + (e_{(b_f)})_b|}{1+|(e_{(a_f)})_a + (e_{(b_f)})_b|} \leq \frac{|(e_{(a_f)})_a|}{1+|(e_{(a_f)})_a|} \leq \frac{|(e_{(a_f)})_a|}{1+|(e_{(a_f)})_a|} + \frac{|(e_{(b_f)})_b|}{1+|(e_{(b_f)})_b|}.$$

This completes the proof. □

THEOREM 3.4. *Suppose three soft points* $(e_{(x_f)})_x$, $(e_{(y_f)})_y$, *and* $(e_{(z_f)})_z$ *satisfy the following inequality:*

$$\frac{|(e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z|}{1 + |(e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z|} \leq \frac{|(e_{(x_f)})_x|}{1 + |(e_{(x_f)})_x|} + \frac{|(e_{(y_f)})_y|}{1 + |(e_{(y_f)})_y|} + \frac{|(e_{(z_f)})_z|}{1 + |(e_{(z_f)})_z|}.$$

PROOF. Let the three soft points $(e_{(x_f)})_x$, $(e_{(y_f)})_y$, and $(e_{(z_f)})_z$ have the same sign. Without loss of generality, we may assume that $(e_{(x_f)})_x \geq 0$, $(e_{(y_f)})_y \geq 0$, and $(e_{(z_f)})_z \geq 0$.

Now,

$$\frac{|(e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z|}{1 + |(e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z|} = \frac{(e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z}{1 + (e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z}$$

$$= \frac{(e_{(x_f)})_x}{1 + (e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z} + \frac{(e_{(y_f)})_y}{1 + (e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z} +$$

$$\frac{(e_{(z_f)})_z}{1 + (e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z}$$

$$\leq \frac{(e_{(x_f)})_x}{1 + (e_{(x_f)})_x} + \frac{(e_{(y_f)})_y}{1 + (e_{(y_f)})_y} + \frac{(e_{(z_f)})_z}{1 + (e_{(z_f)})_z}$$

$$= \frac{|(e_{(x_f)})_x|}{1 + |(e_{(x_f)})_x|} + \frac{|(e_{(y_f)})_y|}{1 + |(e_{(y_f)})_y|} + \frac{|(e_{(z_f)})_z|}{1 + |(e_{(z_f)})_z|}.$$

Suppose now that the three soft points $(e_{(x_f)})_x$, $(e_{(y_f)})_y$, and $(e_{(z_f)})_z$ have different signs. We may assume that $|(e_{(x_f)})_x| \geq |(e_{(y_f)})_y|$ and $|(e_{(y_f)})_y| \geq |(e_{(z_f)})_z|$. Then,

$$|(e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z| \leq |(e_{(x_f)})_x|,$$
$$|(e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z| \leq |(e_{(y_f)})_y|,$$
$$|(e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z| \leq |(e_{(z_f)})_z|.$$

It follows from the above results that:

$$\frac{|(e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z|}{1 + |(e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z|} \leq \frac{|(e_{(x_f)})_x|}{1 + |(e_{(x_f)})_x|} + \frac{|(e_{(y_f)})_y|}{1 + |(e_{(y_f)})_y|} \quad (1)$$

$$\frac{|(e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z|}{1 + |(e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z|} \leq \frac{|(e_{(y_f)})_y|}{1 + |(e_{(y_f)})_y|} + \frac{|(e_{(z_f)})_z|}{1 + |(e_{(z_f)})_z|} \quad (2)$$

From (1) and (2), we have:

$$\frac{|(e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z|}{1 + |(e_{(x_f)})_x + (e_{(y_f)})_y + (e_{(z_f)})_z|} \leq \frac{|(e_{(x_f)})_x|}{1 + |(e_{(x_f)})_x|} + \frac{|(e_{(y_f)})_y|}{1 + |(e_{(y_f)})_y|} + \frac{|(e_{(z_f)})_z|}{1 + |(e_{(z_f)})_z|}$$

This completes the proof.

□

Application 3.1 (Medical Diagnosis)

Patient symptoms under different medical tests (Blood Test) can be modeled as soft points. Suppose U represents the set of Blood Tests and E represents the set of parameters. Let
E = {Blood Chemistry, Complete Blood Count (CBC),
Inflammatory Markers (CRP and ESR), Thyroid Function Test, Hormone Test}.

Suppose that there are five parameters in the blood test given by

$$U = \{(e_{(1_f)})_1, (e_{(2_f)})_2, (e_{(3_f)})_3, (e_{(4_f)})_4, (e_{(5_f)})_5\}$$

and

$$E = \{(e_{(1_b)})_1, (e_{(2_b)})_2, (e_{(3_b)})_3, (e_{(4_b)})_4\}.$$

Where:

$$(e_{(1_b)})_1 = \text{Blood Chemistry}, \quad (e_{(2_b)})_2 = \text{Complete Blood Count (CBC)}$$

$$(e_{(3_b)})_3 = \text{Inflammatory Markers (CRP and ESR)},$$

$$(e_{(4_b)})_4 = \text{Thyroid Function Test}, \quad (e_{(5_b)})_5 = \text{Hormone Test}.$$

Suppose that:

$$F(e_{(1_b)})_1 \leq \{(e_{(1_f)})_1, (e_{(2_f)})_2\},$$

$$F(e_{(2_b)})_2 \leq \{(e_{(3_f)})_3, (e_{(2_f)})_2\},$$

$$F(e_{(3_b)})_3 \leq \{(e_{(1_f)})_1, (e_{(4_f)})_4\},$$

$$F(e_{(4_b)})_4 \leq \{(e_{(4_f)})_4, (e_{(5_f)})_5\},$$

$$F(e_{(5_b)})_5 \leq \{(e_{(5_f)})_5\}.$$

The soft inequality $(F, E, \leq)$ is a parameterized family $\{F(e_i), i = 1, 2, 3, \dots\}$ of subsets of the set U, and it gives the collection of different blood tests. Thus, we can view the soft inequality $(F, E, \leq)$ as a collection of approximations as follows:

$(F, E, \leq)$ = {Blood Chemistry $\leq \{(e_{(1_f)})_1, (e_{(2_f)})_2\}$, Complete Blood Count (CBC) $\leq \{(e_{(3_f)})_3, (e_{(2_f)})_2\}$,

Inflammatory Markers (CRP and ESR) $\leq \{(e_{(1_f)})_1, (e_{(4_f)})_4\}$, Thyroid Function Test $\leq \{(e_{(4_f)})_4, (e_{(5_f)})_5\}$,

Hormone Test $\leq \{(e_{(5_f)})_5\}$}.

Here it is clear that each approximation has two parts: the first part is the established name, and the second part is an approximate value-set. For example, the approximation "Thyroid Function Test $\leq \{(e_{(4_f)})_4, (e_{(5_f)})_5\}$" has the established name "Thyroid Function Test" and the approximate value set $\{(e_{(4_f)})_4, (e_{(5_f)})_5\}$.

In this case, to define a soft inequality $(F, E, \leq)$ with respect to soft points that represent blood test reports, let our universal set contain only two elements, yes and no, such that:

$$U = \{(e_{(y_b)})_y, (e_{(n_b)})_n\}.$$

Now, the ill person's blood test reports having Blood Chemistry, Complete Blood Count (CBC), and so on. After taking the reports, we can construct his soft inequality set $(F, E, \leq)$, which describes whether the ill person is normal or critical.

If the value set result is less than or equal to the result of the blood test, then the illness situation is normal and is represented by "Yes". If the result is more than the given condition, then the illness situation is complicated and is represented by "No".

Application 3.2

Let $\tilde{X} = \mathbb{R}^2$, and for $(e_{(x_f)})_x, (e_{(y_f)})_y \in \mathbb{R}^2$, define $\tilde{d}((e_{(x_f)})_x, (e_{(y_f)})_y)$ by

$$\tilde{d}((e_{(x_f)})_x, (e_{(y_f)})_y) = d\left(\left((e_x)_{(1_f)}(x_1), (e_x)_{(2_f)}(x_2)\right), \left((e_y)_{(1_f)}(y_1), (e_y)_{(2_f)}(y_2)\right)\right)$$

$$= \begin{cases} \left|(e_x)_{(1_f)}(x_1) - (e_y)_{(1_f)}(y_1)\right| & \text{if } (e_x)_{(2_f)}(x_2) = (e_y)_{(2_f)}(y_2) \\ \left|(e_x)_{(1_f)}(x_1)\right| + \left|(e_x)_{(2_f)}(x_2) - (e_y)_{(2_f)}(y_2)\right| + \left|(e_y)_{(1_f)}(y_1)\right| & \text{otherwise} \end{cases}$$

Show that $(\tilde{X}, \tilde{d})$ is a soft metric space.

Proof - Let three elements in $\tilde{X}$ such that

$$(e_{x_f})_x = \left((e^{(1)}_{x_f})_{x_1}, (e^{(2)}_{x_f})_{x_2}\right),$$

$$(e_{y_f})_y = \left((e^{(1)}_{y_f})_{y_1}, (e^{(2)}_{y_f})_{y_2}\right) \quad \text{and} \quad (e_{z_f})_z = \left((e^{(1)}_{z_f})_{z_1}, (e^{(2)}_{z_f})_{z_2}\right).$$

We firstly see here

$$|(e^{(1)}_{x_f})_{x_1} - (e^{(1)}_{y_f})_{y_1}| \leq \tilde{d}((e_{x_f})_x, (e_{y_f})_y).$$

Case 1: If $(e^{(2)}_{x_f})_{x_2} = (e^{(2)}_{y_f})_{y_2}$, then

$$\tilde{d}((e_{x_f})_x, (e_{y_f})_y) = |(e^{(1)}_{x_f})_{x_1} - (e^{(1)}_{y_f})_{y_1}|$$

$$\leq |(e^{(1)}_{x_f})_{x_1} - (e^{(1)}_{z_f})_{z_1}| + |(e^{(1)}_{z_f})_{z_1} - (e^{(1)}_{y_f})_{y_1}|$$

$$\leq \tilde{d}((e_{x_f})_x, (e_{z_f})_z) + \tilde{d}((e_{z_f})_z, (e_{y_f})_y)$$

Case 2: If $(e^{(2)}_{x_f})_{x_2} \neq (e^{(2)}_{y_f})_{y_2}$, then $(e^{(2)}_{z_f})_{z_2}$ cannot be equal to both $(e^{(2)}_{x_f})_{x_2}$ and $(e^{(2)}_{y_f})_{y_2}$. So assume $(e^{(2)}_{z_f})_{z_2} \neq (e^{(2)}_{x_f})_{x_2}$. Then

$$\tilde{d}((e_{x_f})_x, (e_{y_f})_y) = |(e^{(1)}_{x_f})_{x_1}| + |(e^{(2)}_{x_f})_{x_2} - (e^{(2)}_{y_f})_{y_2}| + |(e^{(1)}_{y_f})_{y_1}|$$

$$\leq |(e^{(1)}_{x_f})_{x_1}| + |(e^{(2)}_{x_f})_{x_2} - (e^{(2)}_{z_f})_{z_2}| + |(e^{(2)}_{z_f})_{z_2} - (e^{(2)}_{y_f})_{y_2}| + |(e^{(1)}_{y_f})_{y_1}|$$

If $(e^{(2)}_{y_f})_{y_2} = (e^{(2)}_{z_f})_{z_2}$, then

$$\tilde{d}((e_{x_f})_x, (e_{y_f})_y) = |(e^{(1)}_{x_f})_{x_1}| + |(e^{(2)}_{x_f})_{x_2} - (e^{(2)}_{z_f})_{z_2}| + |(e^{(1)}_{z_f})_{z_1}| + |(e^{(1)}_{z_f})_{z_1} - (e^{(1)}_{y_f})_{y_1}|$$

If $(e^{(2)}_{y_f})_{y_2} \neq (e^{(2)}_{z_f})_{z_2}$, then

$$\tilde{d}((e_{x_f})_x, (e_{y_f})_y) = |(e^{(1)}_{x_f})_{x_1}| + |(e^{(2)}_{x_f})_{x_2} - (e^{(2)}_{z_f})_{z_2}| + |(e^{(1)}_{z_f})_{z_1}|$$

$$+|(e^{(1)}_{z_f})_{z_1}| + |(e^{(2)}_{z_f})_{z_2} - (e^{(2)}_{y_f})_{y_2}| + |(e^{(1)}_{y_f})_{y_1}|$$

Finally,

$$\tilde{d}((e_{x_f})_x, (e_{y_f})_y) \leq \tilde{d}((e_{x_f})_x, (e_{z_f})_z) + \tilde{d}((e_{z_f})_z, (e_{y_f})_y)$$

4. Conclusion:

The soft set theory plays a very important role in every field of mathematics and also in real life problems. In this paper we have given new basic soft inequality on the soft metric space with soft points and introduced some prepositions based on soft inequality as well as the applications based on real life. We hope that the result given in this paper is very fruitful and helpful for future researchers and in solving for a real life problems.

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