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C-COMPACTNESS IN FERMATEAN FUZZY TOPOLOGY

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Abstract

The studies of Fermatean fuzzy sets and Fermatean fuzzy topology was initiated in the year 2019 and 2022 respectively. The present paper created the concepts of C-compactness, nets and filters in Fermatean fuzzy topological spaces. Several results related to characterizations and properties of fermaten fuzzy C-compactness have been established

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1. Introduction

The fusion of technology and generalized forms of classical sets is very useful to solve many real world complex problems which involve the vague and uncertain information. A classical set is defined by its characteristic function from universe of discourse to two point set{0,1}. Classical set theory is insufficient to handle the complex problems involving vague and uncertain information. To handle the vagueness and uncertainty of complex problems, Zadeh [19] in 1965, created fuzzy sets ($\mathcal{FS}$s) as a generalization of classical sets which characterised by membership function from universe of discourse to closed interval [0,1]. $\mathcal{FS}$ theory is applicable in various areas such as control theory, artificial intelligence, pattern recognition , database system and medical diagnosis. After two years of creation of $\mathcal{FS}$s, Chang [3]initiated the study of fuzzy topology. Now fuzzy topology is established a separated branch of fuzzy mathematics. In the last sixty years sevral generalizations of $\mathcal{FS}$s and fuzzy topology was appeared in the literature. Intuitionistic fuzzy sets ($\mathcal{IFS}$s) introduced by Atanassov[1] is a generalization of $\mathcal{FS}$s characterised by membership and non-membership functions from an universe of discourse to closed interval [0,1] whose sum lies between 0 and 1 for each point of universe of discourse. In 1987 Coker [4], created the notion of intuitionistic fuzzy topology. Coker and his coworkers [5, 6], Thakur and his associates [13–16], Lupianez [8] and others are contributed in the development of intuitionistic fuzzy topology. In 2013 Yager [18] introduced Pythagorean fuzzy sets ($\mathcal{PFS}$s) characterized by a membership degree and a non-membership degree whose square sum is less than or equal to one. The collection of

Table 1. Abbreviations and their descriptions

Abbreviation	Description	Abbreviation	Description
$\mathcal{FS}$	Fuzzy set	$\mathcal{IFS}$	Intuitionistic fuzzy set
$\mathcal{PFS}$	Pythagorean fuzzy set	$\mathcal{FFS}$	Fermatean fuzzy set
$\mathcal{FFS}(\mathbb{P})$	Family of $\mathcal{FFS}$s of $\mathbb{P}$	$\mathcal{FFT}$	Fermatean fuzzy topology
$\mathcal{FFTS}$	Fermatean fuzzy topological space	$\mathcal{FFO}$	Fermatean fuzzy open
$\mathcal{FFC}$	Fermatean fuzzy closed	$\mathcal{FFC}(\mathbb{P})$	Family of $\mathcal{FFC}$ sets of $\mathbb{P}$
$\mathcal{FFRO}$	Fermatean fuzzy regular open	$\mathcal{FFRO}(\mathbb{P})$	Family of $\mathcal{FFRO}$ sets of $\mathbb{P}$
$\mathcal{FFRC}$	Fermatean fuzzy regular closed	$\mathcal{FFRCS}(\mathbb{P})$	Family of $\mathcal{FFRC}$ sets of $\mathbb{P}$
$\mathcal{FFP}$	Fermatean fuzzy point	$\mathcal{FFFB}$	Fermatean fuzzy filter base
$\mathcal{FFN}$	Fermatean fuzzy net	$\mathcal{FFA}$	Fermatean fuzzy adherent
$\mathcal{FFSS}$	Fermatean fuzzy sub space	$\mathcal{FFC}$-compact	Fermatean fuzzy C-compact

all $\mathcal{PFS}$s on a universe of discourse contains the collection of all $\mathcal{IFS}$s, but reverse containments is not true. Obviously, $\mathcal{PFS}$s are more effective than $\mathcal{IFS}$s. Peng and Yang [10], studied Some more results for $\mathcal{PFS}$s. In 2019, Olgun and his coworkers [9] introduced pythagorean fuzzy topological spaces and studied continuity and some important pythagorean fuzzy topological concepts. In 2020 Senapati and Yager[11] created the concept of Fermatean fuzzy sets ($\mathcal{FFS}$s) as a generalization of $\mathcal{PFS}$s . In another paper they [12] defined some new operations over $\mathcal{FFS}$s and presented their applications in multi-criteria decision making. Recently Ibrahim [2] created fermaten fuzzy topological spaces as an extension of pythagorean fuzzy topological spaces and studied Fermatean fuzzy continuity and Fermatean fuzzy separation axioms. The study of C-compactness in topology wae initiated by Viglino [17] in 1969. Herrington and Long [7] gave some characterizations of C-compact spaces. The organization of paper is as follows. Section 2 is preliminary and review the basic concepts of Fermatean fuzzy sets and Fermatean fuzzy topology. Section 3 created Fermatean fuzzy nets and Fermatean fuzzy filters and studied their r-convergence. Section 4 defined and characterized Fermatean fuzzy C-compact spaces.

2. Preliminaries

Definition 2.1. Let $\mathbb{P}$ be an universal set. An structure of the form $\mathcal{M} = \{<\mathrm{p}, \varrho_{\mathcal{M}}(\mathrm{p}), \sigma_{\mathcal{M}}(\mathrm{p}) >: \mathrm{p} \in \mathbb{P}\}$ is called:

(a) Intuitionistic fuzzy set($\mathcal{IFS}$)[1] if $0 \leq \varrho_{\mathcal{M}}(\mathrm{p}) + \sigma_{\mathcal{M}}(\mathrm{p}) \leq 1 \;\; \forall \mathrm{p} \in \mathbb{P}$;

(a) Pythagorean fuzzy set($\mathcal{PFS}$)[18] if $0 \leq \varrho_{\mathcal{M}}(\mathrm{p}) + \sigma_{\mathcal{M}}(\mathrm{p}) \leq 1 \;\; \forall \mathrm{p} \in \mathbb{P}$;

(a) Fermatean fuzzy set($\mathcal{FFS}$)[11] if $0 \leq \varrho_{\mathcal{M}}(\mathrm{p}) + \sigma_{\mathcal{M}}(\mathrm{p}) \leq 1 \;\; \forall \mathrm{p} \in \mathbb{P}$.

Where $\varrho_{\mathcal{M}} : \mathbb{P} \to [0, 1]$ and $\sigma_{\mathcal{M}} : \mathbb{P} \to [0, 1]$ are respectively called the membership and non membership function of $\mathcal{M}$. The collection of all $\mathcal{FFS}$s of $\mathbb{P}$ will be denoted by $\mathcal{FFS}(\mathbb{P})$.

Remark 2.2. *[11] Every $\mathcal{IFS}$ is a $\mathcal{PFS}$ and every $\mathcal{PFS}$ is a $\mathcal{FFS}$, but the converse may not be true.*

Definition 2.3. [11] Let $\mathbb{P}$ be an universe of discourse and $\mathcal{M} = \{< p, \varrho_{\mathcal{M}}(p), \sigma_{\mathcal{M}}(p) >: p \in \mathbb{P}\}, \mathcal{N} = \{< p, \varrho_{\mathcal{N}}(p), \sigma_{\mathcal{N}}(p) >: p \in \mathbb{P}\} \in \mathcal{FFS}(\mathbb{P})$. Then :

(a) $\mathcal{M} \subseteq \mathcal{N}$ if $\varrho_{\mathcal{M}}(p) \leq \varrho_{\mathcal{N}}(p)$ and $\sigma_{\mathcal{M}}(p) \geq \sigma_{\mathcal{N}}(p)$ $\forall p \in \mathbb{P}$.
(b) $\mathcal{M} = \mathcal{N}$ if $\mathcal{M} \subseteq \mathcal{N}$ and $\mathcal{N} \subseteq \mathcal{M}$.
(c) $\mathcal{M}^c = \{< p, \sigma_{\mathcal{M}}(p), \varrho_{\mathcal{M}}(p) >: p \in \mathbb{P}\}$.
(d) $\mathcal{M} \cap \mathcal{N} = \{< p, \varrho_{\mathcal{M}}(p) \wedge \varrho_{\mathcal{N}}(p), \sigma_{\mathcal{M}}(p) \vee \sigma_{\mathcal{N}}(p) >: p \in \mathbb{P}\}$
(e) $\mathcal{M} \cup \mathcal{N} = \{< p, \varrho_{\mathcal{M}}(p) \vee \varrho_{\mathcal{N}}(p), \sigma_{\mathcal{M}}(p) \wedge \sigma_{\mathcal{N}}(p) >: p \in \mathbb{P}\}$.
(f) $\tilde{0} = \{< p, 0, 1 >: p \in \mathbb{P}\}$.
(g) $\tilde{1} = \{< p, 1, 0 >: p \in \mathbb{P}\}$.

Definition 2.4. [11] Let $\mathbb{P}$ be an universe of discourse and $\{\mathcal{M}_k : k \in \Lambda\} \subseteq \mathcal{FFS}(\mathbb{P})$. Then:

(a) $\cap\mathcal{M}_k = \{< p, \bigwedge \varrho_{\mathcal{M}_k}(p), \bigvee \sigma_{\mathcal{M}_k}(p) >: p \in \mathbb{P}\}$;
(b) $\cup\mathcal{M}_k = \{< p, \bigvee \varrho_{\mathcal{M}_k}(p), \bigwedge \sigma_{\mathcal{M}_k}(p) >: p \in \mathbb{P}\}$.

Definition 2.5. [2] A collection $\Omega \subseteq \mathcal{FFS}(\mathbb{P})$ is called a Fermatean fuzzy topology ($\mathcal{FFT}$) on $\mathbb{P}$ if:

(1) $\tilde{0}, \tilde{1} \in \Omega$.
(2) $\mathcal{G}_1, \mathcal{G}_2 \in \Omega \Rightarrow \mathcal{G}_1 \cap \mathcal{G}_2 \in \Omega$.
(3) $\{G_\alpha : \alpha \in \Lambda\} \subseteq \Omega \Rightarrow \cup_{\alpha\in\Lambda}\{\mathcal{G}_\alpha : \alpha \in \Lambda\} \in \Omega$.

The structure $(\mathbb{P}, \Omega)$ is called a Fermatean fuzzy topological space ($\mathcal{FFTS}$) and each $\mathcal{FFS}$ in Ω is called Fermatean fuzzy open ($\mathcal{FFO}$) set in $\mathbb{P}$. A $\mathcal{FFS}$ $\mathcal{M}$ is said to be Fermatean fuzzy closed ($\mathcal{FFC}$) if $\mathcal{M}^c \in \Omega$. The collection of all $\mathcal{FFCS}$ sets in a $\mathcal{FFTS}$ $(\mathbb{P}, \Omega)$ is denoted by $\mathcal{FFCS}(\mathbb{P})$.

Definition 2.6. [2] Let $(\mathbb{P}, \Omega)$ be a $\mathcal{FFTS}$ and $\mathcal{M} \in \mathcal{FFS}(\mathbb{P})$. Then the interior and closure of $\mathcal{M}$ are defined by:

$$Cl(\mathcal{M}) = \cap\{\mathcal{F} : \mathcal{F} \in \mathcal{FFCS}(\mathbb{P}) \text{ and } \mathcal{M} \subset \mathcal{F}\}.$$
$$Int(\mathcal{M}) = \cup\{\mathcal{H} : \mathcal{H} \in \Omega \text{ and } \mathcal{H} \subseteq \mathcal{M}\}.$$

Theorem 2.7. *[2] Let* $(\mathbb{P}, \Omega)$ *be a* $\mathcal{FFTS}$ *and* $\mathcal{M} \in \mathcal{FFS}(\mathbb{P})$. *Then:*

(a) $\mathcal{M} \in \mathcal{FFCS}(\mathbb{P}) \Leftrightarrow Cl(\mathcal{M}) = \mathcal{M}$.
(b) $\mathcal{M} \in \Omega \Leftrightarrow Int(\mathcal{M}) = \mathcal{M}$.
(c) $Cl(\mathcal{M}^c) = (Int(\mathcal{M}))^c$.
(d) $Int(\mathcal{M}^c) = (Cl(\mathcal{M}))^c$.

Definition 2.8. [2] Let $\mathbb{P}$ be a non-empty set and $p \in \mathbb{P}$ a fixed element in $\mathbb{P}$. Suppose $\zeta \in (0, 1]$ and $\xi \in [0, 1)$ are two fixed real numbers such that $\zeta^3 + \xi^3 \leq 1$. Then, a Fermatean fuzzy point ($\mathcal{FFP}$) $x^p_{(\zeta,\xi)} = \{\langle p, \varrho_x(p), \sigma_x(p)\rangle\}$ is defined to be a $\mathcal{FFS}$ of $\mathbb{P}$ given by

$$x^p_{(\zeta,\xi)}(q) = \begin{cases} (\zeta, \xi) & \text{if } q = p \\ (0, 1) & \text{otherwise} , \end{cases}$$

for $q \in \mathbb{P}$. In this case, p is called the support of $x^p_{(\zeta,\xi)}$. A $\mathcal{FFP}$ $x^p_{(\zeta,\xi)}$ is said to belong to a $\mathcal{FFS}$ $\mathcal{F} = \{\langle p, \varrho_{\mathcal{F}}(p), \sigma_{\mathcal{F}}(p)\rangle\}$ of $\mathbb{P}$ denoted by $x^p_{(\zeta,\xi)} \in \mathcal{F}$ if $\zeta \leq \varrho_{\mathcal{F}}(p)$ and $\xi \geq \sigma_{\mathcal{F}}(p)$. Two $\mathcal{FFP}$s are said to be distinct if their supports are distinct. The set of all $\mathcal{FFP}$s of $\mathbb{P}$ will be denoted by $\mathcal{FFP}(\mathbb{P})$.

Theorem 2.9. *[2] Let $\mathcal{M}_1 = \{\langle p, \varrho_{\mathcal{M}_1}(p), \sigma_{\mathcal{M}_1}(p)\rangle\}$ and $\mathcal{M}_2 = \{\langle p, \varrho_{\mathcal{M}_2}(p), \sigma_{\mathcal{M}_2}(p)\rangle\}$ be two $\mathcal{FFS}$s of $\mathbb{P}$. Then, $\mathcal{M}_1 \subset \mathcal{M}_2$ if and only if $x^p_{(\zeta,\xi)} \in \mathcal{M}_1$ implies $x^p_{(\zeta,\xi)} \in \mathcal{M}_2$ for any $\mathcal{FFP}$ $x^p_{(\zeta,\xi)}$ in $\mathbb{P}$.*

Definition 2.10. Two $\mathcal{FFS}$ s $\mathcal{M}$ and $\mathcal{N}$ of $\mathbb{P}$ are said to be q-coincident ($\mathcal{M}_q\mathcal{N}$) if $\exists$ an element $p \in \mathbb{P}$ such that $\varrho_{\mathcal{M}}(p) > \sigma_{\mathcal{N}}(p)$ or $\sigma_{\mathcal{M}}(p) < \varrho_{\mathcal{N}}(p)$.

Theorem 2.11. *If $\mathcal{M}$ is a crisp set and $\mathcal{N}$ is any $\mathcal{FFS}$ of a non empty set $\mathbb{P}$. Then $\mathcal{M} \cap \mathcal{N} = \tilde{0} \Leftrightarrow \mathcal{M} \subset \mathcal{N}^c$.*

Definition 2.12. [2] Let $\mathbb{P}$ and $\mathbb{Q}$ be two non-empty sets and $\varphi : \mathbb{P} \to \mathbb{Q}$ be a mapping. Let $\mathcal{M}$ and $\mathcal{N}$ be $\mathcal{FFS}$s of $\mathbb{P}$ and $\mathbb{Q}$, respectively. Then:

(a) The membership and non-membership functions of image of $\mathcal{M}$ with respect to φ that is denoted by $\varphi(\mathcal{M})$ are defined by

$$\varrho_{\varphi(\mathcal{M})}(q) = \begin{cases} \sup\limits_{r\in\varphi^{-1}(q)} \varrho_{\mathcal{M}}(r) & \text{if } \varphi^{-1}(q) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

and

$$\sigma_{\varphi(\mathcal{M})}(q) = \begin{cases} \inf\limits_{r\in\varphi^{-1}(q)} \sigma_{\mathcal{M}}(r) & \text{if } \varphi^{-1}(q) \neq \phi \\ 1 & \text{otherwise} \end{cases}$$

respectively.

(b) The membership and non-membership functions of pre-image of $\mathcal{N}$ with respect to φ that is denoted by $\varphi^{-1}(\mathcal{N})$ are respectively defined by $\varrho_{\varphi^{-1}(\mathcal{N})}(p) = \varrho_{\mathcal{N}}(\varphi(p))$ and $\sigma_{\varphi^{-1}(\mathcal{N})}(p) = \sigma_{\mathcal{N}}(\varphi(p))$.

Definition 2.13. [2] A mapping $\varphi : (\mathbb{P}, \Omega \to (\mathbb{Q}, \Gamma)$ is called Fermatean fuzzy continuous if $\varphi^{-1}(\mathcal{M}) \in \Omega, \ \forall \mathcal{M} \in \Gamma$.

3. Fermatean fuzzy filters

Definition 3.1. Let $\mathbb{P}$ be an universe of discourse. A nonempty family $\mathfrak{F} \subseteq \mathcal{FFS}(\mathbb{P})$ is called a Fermatean fuzzy filter base ($\mathcal{FFFB}$) in $\mathbb{P}$ if:

(a) $\tilde{0} \notin \mathfrak{F}$;

(b) $\mathcal{F}_1 \cap \mathcal{F}_2 \in \mathfrak{F}, \forall \ \mathcal{F}_1, \mathcal{F}_2 \in \mathfrak{F}$;

(c) $\mathcal{F}_1 \in \mathfrak{F}$ and $\mathcal{F}_1 \subseteq \mathcal{F}_2 \Rightarrow \mathcal{F}_2 \in \mathfrak{F}, \ \forall \mathcal{F}_1, \mathcal{F}_2 \in \mathfrak{F}$.

Definition 3.2. Let $(\mathbb{P}, \Omega)$ be a $\mathcal{FFTS}$ and $\mathcal{N} \in \mathcal{FFS}(\mathbb{P})$. Then $\mathcal{N}$ is called an ϵ-neighbourhood of a $\mathcal{FFP}$ $x^p_{(\zeta,\xi)}$ of $\mathbb{P}$ if $\exists$ $\mathcal{G} \in \Omega$ such that $x^p_{(\zeta,\xi)} \in \mathcal{G} \subseteq \mathcal{N}$. The family of all an ϵ neighbourhood of $x^p_{(\zeta,\xi)}$ is denoted by $\mathfrak{N}(x^p_{(\zeta,\xi)})$.

Definition 3.3. Let $\mathbb{P}$ be a non empty set and $\mathbb{D}$ is a directed set. A map $\phi : \mathbb{D} \to \mathcal{FFP}(\mathbb{P})$ is called a Fermatean fuzzy net($\mathcal{FFN}$). We will write $\phi_d = \phi(d)$(for $d \in \mathbb{D}$), $\phi = (\phi_d)_{d\in\mathbb{D}}$.

Definition 3.4. Let $(\mathbb{P}, \Omega)$ be a $\mathcal{FFTS}$ and $\mathcal{M}$ be a non empty crisp set of $\mathbb{P}$, let $\mathfrak{F} = \{M_\alpha \subset \mathcal{M} : \alpha \in \Delta\}$ be a $\mathcal{FFFB}$ in $\mathcal{M}$ Then :

(a) $\mathcal{FFFB}$ $\mathfrak{F}$ is called r-converges to a $\mathcal{FFP}$ $x^p_{(\zeta,\xi)} \in \mathcal{M}$ (written as $\mathfrak{F} \underset{r}{\to} x^p_{(\zeta,\xi)}$), if $\forall\ \mathcal{N} \in \mathfrak{N}(x^p_{(\zeta,\xi)})\ \exists\ \mathcal{M}_\alpha \in \mathfrak{F}$ such that $\mathcal{M}_\alpha \subset Cl(\mathcal{N})$.

(b) $\mathcal{FFFB}$ $\mathfrak{F}$ is called r-accumulates to a $\mathcal{FFP}$ $x^p_{(\zeta,\xi)} \in \mathcal{M}$ (written as $\underset{r}{\propto} x^p_{(\zeta,\xi)}$), if $\forall\ \mathcal{N} \in \mathfrak{N}(x^p_{(\zeta,\xi)})$ and each $\mathcal{M}_\alpha \in \mathfrak{F}$, $\quad \mathcal{M}_\alpha \cap Cl(\mathcal{N}) \neq \tilde{0}$.

Theorem 3.5. *Let* $(\mathbb{P}, \Omega)$ *be a* $\mathcal{FFTS}$, $\mathcal{M} \subset \mathbb{P}$ *and* $x^p_{(\zeta,\xi)} \in \mathcal{FFP}(\mathbb{P})$.

(a) *Let* $\mathfrak{F}$ *is a* $\mathcal{FFFB}$ *in* $\mathcal{M}$ *. If* $\mathfrak{F} \underset{r}{\to} x^p_{(\zeta,\xi)} \in \mathcal{M}$, *then* $\mathfrak{F} \underset{r}{\propto} x^p_{(\zeta,\xi)}$.

(b) *Let* $\mathfrak{F}_1$ *and* $\mathfrak{F}_2$ *be two* $\mathcal{FFFB}$ *in* $\mathcal{M}$ *and* $\mathfrak{F}_2$ *is stronger than* $\mathfrak{F}_1(\mathfrak{F}_1 \subset \mathfrak{F}_2)$. *If* $\mathfrak{F}_2 \underset{r}{\propto} x^p_{(\zeta,\xi)} \in \mathcal{M}$, *then* $\mathfrak{F}_1 \underset{r}{\propto} x^p_{(\zeta,\xi)}$.

(c) *Let* $\mathfrak{M}$ *be a maximal* $\mathcal{FFFB}$ *in* $\mathcal{M}$. *Then* $\mathfrak{F} \underset{r}{\propto} x^p_{(\zeta,\xi)} \in \mathcal{M} \Leftrightarrow \mathfrak{F} \underset{r}{\to} x^p_{(\zeta,\xi)}$.

Proof. Obvious and left to the readers. □

Definition 3.6. Let $(\mathbb{P}, \Omega)$ be a $\mathcal{FFTS}$ and $\mathcal{M}$ be a non empty crisp subset of $\mathbb{P}$. Suppose $\mathbb{D}$ is a directed set and $\phi : \mathbb{D} \to \mathcal{FFP}(\mathcal{M})$ is a $\mathcal{FFN}$. then:

(a) ϕ is called r-converges to a $\mathcal{FFP}$ $x^p_{(\zeta,\xi)} \in \mathcal{M}$(written as $\phi \underset{r}{\to} x^p_{(\zeta,\xi)}$) if $\forall\ \mathcal{V} \in \Omega$ containing $x^p_{(\zeta,\xi)}$, $\exists\ b \in \mathbb{D}$ such that $\phi(T_b) \subset Cl(\mathcal{V})$. Where $\mathcal{T}_b = \{c \in \mathbb{D} : c \geq b\}$.

(b) ϕ called r-accumulates to a $\mathcal{FFP}$ $x^p_{(\zeta,\xi)} \in \mathcal{M}$ (written as $\phi \underset{r}{\propto} x^p_{(\zeta,\xi)}$) if $\forall\ \mathcal{V} \in \Omega$ containing $x^p_{(\zeta,\xi)}$, and $\forall\ b \in \mathbb{D}$ such that $\phi(\mathcal{T}_b) \cap Cl(\mathbb{V}) \neq \tilde{0}$. Where $\mathcal{T}_b = \{c \in \mathbb{D} : c \geq b\}$.

Definition 3.7. If $\phi : \mathbb{D} \to \mathcal{FFP}(\mathcal{M})$ is a $\mathcal{FFN}$ in $\mathcal{M} \subset \mathbb{P}$. Then the family $\mathfrak{F}(\phi) = \{\phi(\mathcal{T}_b) : c \in \mathbb{D}\}$ is a $\mathcal{FFFB}$ in $\mathcal{M}$ called a $\mathcal{FFFB}$ associated to ϕ.

Theorem 3.8. *Let* $(\mathbb{P}, \Omega)$ *be a* $\mathcal{FFTS}$. *If* $\mathfrak{F}(\phi)$ *is a* $\mathcal{FFFB}$ *in* $\mathcal{M} \subset \mathbb{P}$ *associated to* $\mathcal{FFN}$ ϕ. *Then*

(a) $\mathfrak{F}(\phi) \underset{r}{\to} x^p_{(\zeta,\xi)} \in \mathcal{M} \Leftrightarrow \phi \underset{r}{\to} x^p_{(\zeta,\xi)}$.

(b) $\mathfrak{F}(\phi) \underset{r}{\propto} x^p_{(\zeta,\xi)} \in \mathcal{M} \Leftrightarrow \phi \underset{r}{\propto} x^p_{(\zeta,\xi)}$.

Proof. Obvious. □

Definition 3.9. Let $\mathbb{P}$ be a non empty set and $\mathcal{M}$ be a crisp subset of $\mathbb{P}$. Let $\mathfrak{F}$ be a $\mathcal{FFFB}$ in $\mathbb{P}$ then consider the family $\mathbb{D}_{\mathfrak{F}} = \{(x^p_{(\zeta,\xi)}, \mathfrak{F}) : x^p_{(\zeta,\xi)} \in \mathcal{FFP}(\mathcal{M}), x^p_{(\zeta,\xi)} \in \mathcal{F}, \mathcal{F} \in \mathfrak{F}\}$ with the relation $(x^p_{(\zeta,\xi)}, \mathcal{F}) \leq (x^{p'}_{(\zeta,\xi)}, \mathcal{F}') \Leftrightarrow \mathcal{F}' \subseteq \mathcal{F}$ .Then the

$\mathcal{FFN}$ $\phi_{\mathfrak{F}} : \mathbb{D}_{\mathfrak{F}} \to \mathcal{FFP}(\mathcal{M})$ such that $\phi_{\mathfrak{F}}(x^p_{(\zeta,\xi)}, \mathcal{F}) = x^p_{(\zeta,\xi)}$ is called called $\mathcal{FFN}$ associate to $\mathfrak{F}$.

THEOREM 3.10. *In a* $\mathcal{FFTS}$ $(\mathbb{P}, \Omega)$, *If* $\phi_{\mathfrak{F}} : \mathbb{D} \to \mathcal{FFP}(\mathcal{M})$ *is a* $\mathcal{FFN}$ *in* $\mathcal{M} \subset \mathbb{P}$ *associated to a* $\mathcal{FFFB}$ $\mathfrak{F}$. *then:*

(a) $\mathfrak{F} \underset{r}{\to} x^p_{(\zeta,\xi)} \in \mathcal{M} \Leftrightarrow \phi_{\mathfrak{F}} \underset{r}{\to} x^p_{(\zeta,\xi)}$.

(b) $\mathfrak{F} \underset{r}{\propto} x^p_{(\zeta,\xi)} \in \mathcal{M} \Leftrightarrow \phi_{\mathfrak{F}} \underset{r}{\propto} x^p_{(\zeta,\xi)}$.

PROOF. (a)Necessity. Since $\mathfrak{F}$ r-converges to $x^p_{(\zeta,\xi)}$ in $\mathcal{M}$, $\forall$ $\mathcal{N} \in \mathfrak{R}(x^p_{(\zeta,\xi)})$ $\exists$ $\mathcal{M}_\alpha \in \mathfrak{F}$ such that $\mathcal{M}_\alpha \subset Cl(\mathcal{N})$. For every $\mathcal{FFP}$ $x^q_{(\zeta,\xi)}$ such that $x^q_{(\zeta,\xi)} \in \mathcal{M}_\alpha$, $(x^q_{(\zeta,\xi)}, \mathcal{M}_\alpha) \in \mathbb{D}_{\mathfrak{F}}$. If$(x^{q'}_{(\zeta,\xi)}, \mathcal{M}_\beta) \in \mathbb{D}_{\mathfrak{F}}$ and $(x^{q'}_{(\zeta,\xi)}, \mathcal{M}_\beta) \geq (x^q_{(\zeta,\xi)}, \mathcal{M}_\alpha)$ then $x^{q'}_{(\zeta,\xi)} \in \mathcal{M}_\beta$ and $\mathcal{M}_\beta \subset \mathcal{M}_\alpha$.Thus $\phi_{\mathfrak{F}}(x^{q'}_{(\zeta,\xi)}, \mathcal{M}_\beta) = x^{q'}_{(\zeta,\xi)} \in \mathcal{M}_\alpha \subset Cl(\mathcal{N})$.Hence $\phi_{\mathfrak{F}} \underset{r}{\to} x^p_{(\zeta,\xi)}$.

Sufficiency. Since $\phi_{\mathfrak{F}}$ r-converges to $x^p_{(\zeta,\xi)}$ in $\mathcal{M}$, $\forall$ $\mathcal{N} \in \mathfrak{R}(x^p_{(\zeta,\xi)})$ $\exists$ $(x^{q_0}_{(\zeta,\xi)}, \mathcal{M}_0) \in \mathbb{D}_{\mathfrak{F}}$ such that $\phi_{\mathfrak{F}}(x^q_{(\zeta,\xi)}, \mathcal{M}_\alpha) = x^q_{(\zeta,\xi)} \in \mathcal{M}_\alpha \subset Cl(\mathcal{N})$ $\forall$ $(x^q_{(\zeta,\xi)}, \mathcal{M}_\alpha) \geq (x^{q_0}_{(\zeta,\xi)}, \mathcal{M}_0)$. This implies that $\mathcal{M}_0 \subset \mathcal{M}_\alpha$ because $\forall$ $\mathcal{FFP}$ $x^q_{(\zeta,\xi)}$ in $\mathbb{P}$ $x^q_{(\zeta,\xi)} \in \mathcal{M}_0$ we have $(x^q_{(\zeta,\xi)}, \mathcal{M}_0) \geq (x^{q_0}_{(\zeta,\xi)}, \mathcal{M}_0)$, $x^q_{(\zeta,\xi)} \in \mathcal{M}_\alpha$. Consequently, $\mathcal{M}_0 \subset Cl(\mathcal{N})$. Hence, $\mathfrak{F} \underset{r}{\to} x^p_{(\zeta,\xi)} \in \mathcal{M}$.

(b)Necessity. Since $\mathfrak{F}$ r-accumulates to $x^p_{(\zeta,\xi)}$ in $\mathcal{M}$, $\forall$ $\mathcal{N} \in \mathfrak{R}(x^p_{(\zeta,\xi)})$ $\exists$ $\mathcal{M}_\alpha \in \mathfrak{F}$ such that $\mathcal{M}_\alpha \cap Cl(\mathcal{N}) \neq \tilde{0}$. For every $\mathcal{FFP}$ $x^q_{(\zeta,\xi)}$ such that $x^q_{(\zeta,\xi)} \in \mathcal{M}_\alpha$, $(x^q_{(\zeta,\xi)}, \mathcal{M}_\alpha) \in \mathbb{D}_{\mathfrak{F}}$. If$(x^{q'}_{(\zeta,\xi)}, \mathcal{M}_\beta) \in \mathbb{D}_{\mathfrak{F}}$ and $(x^{q'}_{(\zeta,\xi)}, \mathcal{M}_\beta) \geq (x^q_{(\zeta,\xi)}, \mathcal{M}_\alpha)$ then $(x^{q'}_{(\zeta,\xi)} \in \mathcal{M}_\beta$ and $\mathcal{M}_\beta \subset \mathcal{M}_\alpha$.Thus $\phi_{\mathfrak{F}}(x^{q'}_{(\zeta,\xi)}, \mathcal{M}_\beta) = x^{q'}_{(\zeta,\xi)} \in \mathcal{M}_\alpha$. This implies that $\phi_{\mathfrak{F}}(x^{q'}_{(\zeta,\xi)}, \mathcal{M}_\beta) \cap Cl(\mathcal{N}) \neq \tilde{0}$. Hence, $\phi_{\mathfrak{F}} \underset{r}{\propto} x^p_{(\zeta,\xi)}$.

Sufficiency. Since $\phi_{\mathfrak{F}}$ r-accumulates to $x^p_{(\zeta,\xi)}$ in $\mathcal{M}$, $\forall$ $\mathcal{N} \in \mathfrak{R}(x^p_{(\zeta,\xi)})$ $\exists$ $(x^{q_0}_{(\zeta,\xi)}, \mathcal{M}_0) \in \mathbb{D}_{\mathfrak{F}}$ such that $\phi_{\mathfrak{F}}(x^q_{(\zeta,\xi)}, \mathcal{M}_\alpha) = x^q_{(\zeta,\xi)} \in \mathcal{M}_\alpha$ and $\phi_{\mathfrak{F}}(x^q_{(\zeta,\xi)}, \mathcal{M}_\alpha) \cap Cl(\mathcal{N}) \neq \tilde{0}$, $\forall$ $(x^q_{(\zeta,\xi)}, \mathcal{M}_\alpha) \geq (x^{q_0}_{(\zeta,\xi)}, \mathcal{M}_0)$. This implies that $\mathcal{M}_0 \subset \mathcal{M}_\alpha$ because $\forall$ $\mathcal{FFP}$ $x^q_{(\zeta,\xi)}$ in $\mathbb{P}$ $x^q_{(\zeta,\xi)} \in \mathcal{M}_0$ we have $(x^q_{(\zeta,\xi)}, \mathcal{M}_0) \geq (x^{q_0}_{(\zeta,\xi)}, \mathcal{M}_0)$, $x^q_{(\zeta,\xi)} \in \mathcal{M}_\alpha$. Consequently, $\mathcal{M}_\alpha \cap Cl(\mathcal{N}) \neq \tilde{0}$. Hence, $\mathfrak{F} \underset{r}{\propto} x^p_{(\zeta,\xi)}$ in $\mathcal{M}$. □

4. Fermatean fuzzy C-compactness

DEFINITION 4.1. A family $\{\mathcal{G}_\alpha : \alpha \in \Lambda\}$ of $\mathcal{FFS}$ s of a $\mathcal{FFTS}$ $(\mathbb{P}, \Omega)$ is called a Fermatean fuzzy cover of $\mathbb{P}$ if $\tilde{1} = \cup_{\alpha\in\Lambda}\{\mathcal{G}_\alpha\}$.

DEFINITION 4.2. A $\mathcal{FFTS}$ $(\mathbb{P}, \Omega)$ is called Fermatean fuzzy compact if every $\mathcal{FFO}$ cover of $\mathbb{P}$ has a finite sub cover.

DEFINITION 4.3. A $\mathcal{FFTS}$ $(\mathbb{P}, \Omega)$ is said to be Fermatean fuzzy C-compact($\mathcal{FFC}$-compact) if $\forall$ proper$\mathcal{FFC}$ crisp set $\mathcal{M}$ of $(\mathbb{P})$ and $\forall$ $\mathcal{FFO}$ cover $\{\mathcal{G}_\alpha : \alpha \in \Lambda\}$ of $\mathcal{M}$, $\exists$ a finite number of elements $\mathcal{G}_{\alpha_1}, \mathcal{G}_{\alpha_2}, \mathcal{G}_{\alpha_3}, \ldots \mathcal{G}_{\alpha_n}$ such that $\mathcal{M} \subset \bigcup_{i=1}^n \{Cl(\mathcal{G}_{\alpha_i})\}$.

Theorem 4.4. *Let* $(\mathbb{P}, \Omega)$ *be a* $\mathcal{FFTS}$ *. If* $\mathbb{P}$ *is Fermatean fuzzy compact then it is* $\mathcal{FFC}$*- compact.*

Proof. Easy and left to the readers. □

Definition 4.5. A $\mathcal{FFS}$ $\mathcal{M}$ in a $\mathcal{FFTS}$ $(\mathbb{P}, \Omega)$ is called:

(a) Fermatean fuzzy regular open ($\mathcal{FFRO}$) if $\mathcal{M} = Int(Cl(\mathcal{M}))$.The collection of all $\mathcal{FFRO}$ sets of $\mathbb{P}$ will be denoted by $\mathcal{FFROS}(\mathbb{P})$.
(b) Fermatean fuzzy regular closed ($\mathcal{FFRC}$) if $\mathcal{M}^c \in \mathcal{FFROS}(\mathbb{P})$.The collection of all $\mathcal{FFRC}$ sets of $\mathbb{P}$ will be denoted by $\mathcal{FFRCS}(\mathbb{P})$.

Remark 4.6. *In a* $\mathcal{FFTS}$ $(\mathbb{P}, \Omega)$ *,* $\mathcal{FFROS}(\mathbb{P}) \subset \Omega$ *and* $\mathcal{FFRCS}(\mathbb{P}) \subset \mathcal{FFCS}(\mathbb{P})$*, but the reverse containment may not be true.*

Example Let $\mathbb{P} = \{p_1, p_2\}$ be an universe of discourse and $\Omega = \{\tilde{0}, \mathcal{M}, \tilde{1}\}$ be a $\mathcal{FFT}$ on $\mathbb{P}$. Where, $\mathcal{M} = \{< p_1, 0.8, 0.7 >, < p_2, 0.7, 0.8 >\}$. Then $\mathcal{M} \in \Omega$ (resp.$\mathcal{M}^c \in \mathcal{FFCS}(\mathbb{P})$) but $\mathcal{M} \notin \mathcal{FFROS}(\mathbb{P})$ (resp. $\mathcal{M}^c \notin \mathcal{FFRCS}(\mathbb{P})$).

Theorem 4.7. *Let* $(\mathbb{P}, \Omega)$ *be a* $\mathcal{FFTS}$ *and* $\mathcal{M} \in \mathcal{FFS}(\mathbb{P})$*. Then* $Int(Cl(\mathcal{M})) \in \mathcal{FFROS}(\mathbb{P})$ *and* $Cl(Int(\mathcal{M})) \in \mathcal{FFRCS}(\mathbb{P})$*.*

Theorem 4.8. *In a* $\mathcal{FFTS}$ $(\mathbb{P}, \Omega)$ *the next statements are equivalent:*

(a) $\mathbb{P}$ *is* $\mathcal{FFC}$*-compact.*
(b) *For each* $\mathcal{FFC}$ *crisp set* $\mathcal{M}$ *of* $\mathbb{P}$ *and each* $\mathcal{FFRO}$ *cover* $\mathcal{G} = \{\mathcal{G}_\alpha : \alpha \in \Lambda\}$ *of* $\mathcal{M}$ $\exists$ *a finite number of elements* $\mathcal{G}_{\alpha_1}, \mathcal{G}_{\alpha_2}, \mathcal{G}_{\alpha_3}, \ldots \mathcal{G}_{\alpha_n}$ *of* $\mathcal{G}$ *such that* $\mathcal{M} \subseteq \bigcup_{i=1}^{n} Cl(\mathcal{G}_{\alpha_i})$*.*
(c) *For each crisp set* $\mathcal{M} \in \mathcal{FFCS}(\mathbb{P})$ *and for each collection* $\mathcal{F} = \{\mathcal{F}_\alpha : \alpha \in \Lambda\}$ *of non empty* $\mathcal{FFRC}$ *sets of* $\mathbb{P}$ *such that* $(\cap_{\alpha\in\Lambda}\mathcal{F}_\alpha) \cap \mathcal{M} = \tilde{0}$*,* $\exists$ *a finite number of elements* $\mathcal{F}_{\alpha_1}, \mathcal{F}_{\alpha_2}, \mathcal{F}_{\alpha_3}, \ldots \mathcal{F}_{\alpha_n}$ *of* $\mathcal{F}$ *such that* $(\bigcap_{i=1}^{n} Int(\mathcal{F}_{\alpha_i})) \cap \mathcal{M} = \tilde{0}$*.*
(d) *For each crisp set* $\mathcal{M} \in \mathcal{FFCS}(\mathbb{P})$ *and for each collection* $\mathcal{F} = \{\mathcal{F}_\alpha : \alpha \in \Lambda\}$ *of* $\mathcal{FFRC}$ *sets of* $\mathbb{P}$*, if for each finite subcollection* $\{\mathcal{F}_{\alpha_1}, \mathcal{F}_{\alpha_2}, \mathcal{F}_{\alpha_3}, \cdots \mathcal{F}_{\alpha_n}\}$*, of* $\mathcal{F}$ *has the property that* $\bigcap_{i=1}^{n}(Int(F_{\alpha_i})) \cap \mathcal{M} \neq \tilde{0}$*, then* $(\cap_{\alpha\in\Lambda}\mathcal{F}_\alpha) \cap \mathcal{M} \neq \tilde{0}$*.*
(e) *For each crisp set* $\mathcal{M} \in \mathcal{FFCS}(\mathbb{P})$ *and each* $\mathcal{FFFB}$ $\mathfrak{F} = \{\mathcal{M}_\alpha : \alpha \in \Lambda\}$ *in* $\mathcal{M}$ $\exists$ *a* $\mathcal{FFP}$ $x^p_{(\zeta,\xi)} \in \mathcal{M}$ *such that* $\mathfrak{F} \underset{r}{\propto} x^p_{(\zeta,\xi)}$*.*
(f) *For each crisp set* $\mathcal{M} \in \mathcal{FFCS}(\mathbb{P})$ *and each maximal* $\mathcal{FFFB}$ $\mathfrak{M} = \{\mathcal{M}_\alpha : \alpha \in \Lambda\}$ *in* $\mathcal{M}$ $\exists$ *a* $\mathcal{FFP}$ $x^p_{(\zeta,\xi)} \in \mathcal{M}$ *such that* $\mathfrak{M} \underset{r}{\propto} x^p_{(\zeta,\xi)}$*.*

Proof. (a) ⇒ (b) Follows easily from Definition 4.3 and Remark 4.6.

(b) ⇒ (a). Suppose (b) holds.Let $\{\mathcal{G}_\alpha : \alpha \in \Lambda\}$ be a $\mathcal{FFO}$ cover of a crisp set $\mathcal{M} \in \mathcal{FFCS}(\mathbb{P})$. Then by thm $\{Int(Cl(\mathcal{G}_\alpha)) : \alpha \in \Lambda\}$. will be $\mathcal{FFRO}$ cover of $\mathcal{M}$. Therefore , by (b) $\exists$ a finite sub collection $\{Int(Cl(\mathcal{G}_{\alpha_i})) : i = 1, 2, 3 \ldots n\}$ such that $\mathcal{M} \subset \bigcup_{i=1}^{n} Cl(Int(Cl(\mathcal{G}_{\alpha_i}))) = \bigcup_{i=1}^{n} Cl(\mathcal{G}_{\alpha_i})$.. Consequently, $\mathcal{M} \subset \bigcup_{i=1}^{n} Cl(\mathcal{G}_{\alpha_i})$ and $\mathbb{P}$ is $\mathcal{FFC}$-compact.

(b) ⇒ (c). Let $\mathcal{M}$ be a crisp $\mathcal{FFCS}$ of $\mathbb{P}$. Let $\mathcal{F} = \{\mathcal{F}_\alpha : \alpha \in \Lambda\}$ be a collection of non empty $\mathcal{FFRC}$ sets of $\mathbb{P}$ such that $(\cap\{\mathcal{F}_\alpha : \alpha \in \Lambda\}) \cap \mathcal{M} = \tilde{0}$ for each proper crisp set $\mathcal{M} \in \mathcal{FFCS}(\mathbb{P})$. Then $\mathcal{U} = \{\mathcal{F}^c_\alpha : \alpha \in \Lambda\}$ is $\mathcal{FFRO}$ cover of crisp set$\mathcal{M}\mathcal{FFCS}(\mathbb{P})$. Therefore $\exists$ a finite sub collection $\{\mathcal{G}_{\alpha_i} = \mathcal{F}^c_{\alpha_i} : i = 1, 2, 3 \ldots n\}$ of $\mathcal{U}$ such that $\mathcal{M} \subset \bigcup_{i=1}^n Cl(\mathcal{G}_{\alpha_i})$. Now for each α_i we have, $Int(\mathcal{F}_{\alpha_i}) = Int(\mathcal{G}^c_{\alpha_i}) = (Cl(\mathcal{G}^c_{\alpha_i})^c)^c = Cl(\mathcal{G}^c_{\alpha_i})$. Therefore $\bigcap_{i=1}^n Int(\mathcal{F}_{\alpha_i}) = \bigcup_{i=1}^n (Cl(\mathcal{G}_{\alpha_i}))^c = \mathcal{M}^c$. This shows that $\bigcap_{i=1}^n (Int(\mathcal{F}_{\alpha_i})) \cap \mathcal{M} = \mathcal{M}^c \cap \mathcal{M} = \tilde{0}$, because $\mathcal{M}$ is a crisp set of $\mathbb{P}$.

(c) $\Rightarrow$ (b). Let$\mathcal{U} = \{\mathcal{G}_\alpha : \alpha \in \Lambda\}$ be a $\mathcal{FFRO}$ cover of a proper crisp set $\mathcal{M} \in \mathcal{FFCS}(\mathbb{P})$. Therefore, $\mathcal{M} \subset \bigcup_{\alpha\in\Lambda} \mathcal{G}_\alpha$. It follows that $\mathcal{M}^c \supseteq (\bigcup_{\alpha\in\Lambda} \mathcal{G}_\alpha)^c = \bigcap_{\alpha\in\Lambda} G_\alpha{}^c$. And so, $(\cap_{\alpha\in\Lambda}\mathcal{G}_\alpha{}^c) \cap \mathcal{M} \subset \mathcal{M}^c \cap \mathcal{M} = \tilde{0}$, because $\mathcal{M}$ is a crisp set of $\mathbb{P}$. Therefore $\mathcal{F} = \{\mathcal{G}^c_\alpha : \alpha \in \Lambda\}$ is a collection of non empty $\mathcal{FFRCS}$s of $\mathbb{P}$, satisfying $(\cap_{\alpha\in\Lambda}\mathcal{F}) \cap \mathcal{M} = \tilde{0}$. And so by (c) , $\exists$ a finite sub collection $\{\mathcal{G}^c_{\alpha_i} : i = 1, 2, 3 \ldots n\}$ of $\mathcal{F}$ such that $\bigcap_{i=1}^n (Int(\mathcal{G}^c_{\alpha_i})) \cap \mathcal{M} = \tilde{0}$. It follows that, $\mathcal{M} \subset \bigcup_{i=1}^n (Int(\mathcal{G}^c_{\alpha_i}))^c)$. Now for each α_i we have, $Int(\mathcal{G}^c_{\alpha_i}) = (Cl(\mathcal{G}^c_{\alpha_i})^c)^c = Cl(\mathcal{G}_{\alpha_i})^c$. Therefore, we obtain that $\mathcal{M} \subset \bigcup_{i=1}^n Cl(\mathcal{G}_{\alpha_i})$.

(c) $\Rightarrow$ (d). Let $\mathcal{M}$ be a crisp $\mathcal{FFC}$ set of $\mathbb{P}$. Let $\mathcal{F} = \{\mathcal{F}_\alpha : \alpha \in \Lambda\}$ be a collection of non empty $\mathcal{FFRC}$ sets of $\mathbb{P}$ such that for every finite subcollection $\{\mathcal{F}_{\alpha_1}, \mathcal{F}_{\alpha_2}, \mathcal{F}_{\alpha_3},, \ldots \mathcal{F}_{\alpha_n}\}$ }of $\mathcal{F}$ we have $\bigcap_{i=1}^n Int(\mathcal{F}_{\alpha_i}) \cap \mathcal{M} \neq \tilde{0}$. We want to show that $(\cap\mathcal{F}_\alpha) \cap \mathcal{M} \neq \tilde{0}$. If $(\cap\mathcal{F}_\alpha) \cap \mathcal{M} = \tilde{0}$, Then by (c), $\exists$ a finite family $\{\mathcal{F}_{\alpha_1}, \mathcal{F}_{\alpha_2}, \mathcal{F}_{\alpha_3}, \ldots \mathcal{F}_{\alpha_n}\}$., such that $\bigcap_{i=1}^n Int(\mathcal{F}_{\alpha_i}) \cap \mathcal{M} = \tilde{0}$, which is a contradiction. Hence $(\cap\mathcal{F}_\alpha) \cap \mathcal{M} \neq \tilde{0}$.

(d) $\Rightarrow$ (c). Le $\mathcal{M}$ be a crisp $\mathcal{FFC}$ set of $\mathbb{P}$ and $\mathcal{F} = \{\mathcal{F}_\alpha : \alpha \in \Lambda\}$ be a collection of non empty$\mathcal{FFRC}$ sets of $\mathbb{P}$ such that $(\cap\mathcal{F}_{\alpha_i}) \cap \mathcal{M} = \tilde{0}$. We have to show that $\exists$ a finite integer (say) n such that $\bigcap_{i=1}^n int(\mathcal{F}_{\alpha_i}) \cap \mathcal{M} = \tilde{0}$. Suppose now that for every finite integer n we have $\bigcap_{i=1}^n Int(\mathcal{F}_{\alpha_i}) \cap \mathcal{M} \neq \tilde{0}$. Then by (d) we have $(\cap\mathcal{F}_\alpha) \cap \mathcal{M} \neq \tilde{0}$ which is a contradiction.

(a) $\Rightarrow$ (e). Suppose $\exists$ $\mathcal{FFFB}$ $\mathfrak{F} = \{\mathcal{M}_\alpha : \alpha \in \Lambda\}$ in $\mathcal{M}$, such that $\mathfrak{F} \underset{r}{\propto} x^p_{(\zeta,\xi)}$ for all $\mathcal{FFP}$ $x^p_{(\zeta,\xi)} \in \mathcal{M}$. Then $\forall$ $x^p_{(\zeta,\xi)} \in \mathcal{M}$ $\exists$ $\mathcal{N}(x^p_{(\zeta,\xi)}) \in \Omega$ and some $\mathcal{M}_{\alpha(x^p_{(\zeta,\xi)})} \in \mathfrak{F}$ such that $\mathcal{M}_{\alpha(x^p_{(\zeta,\xi)})} \cap Cl(\mathcal{N}(x^p_{(\zeta,\xi)})) = \tilde{0}$. The collection $\{\mathcal{N}(x^p_{(\zeta,\xi)}) : x^p_{(\zeta,\xi)} \in \mathcal{M}\}$ is a $\mathcal{FFO}$ cover of $\mathcal{M}$, so by (a) $\exists$ a finite sub collection $\{\mathcal{N}(x^p_{(\zeta_i,\xi_i)}) : i = 1, 2, 3, \ldots, n\}$ such that $\mathcal{M} \subset \bigcup_{i=1}^n Cl(\mathcal{N}(x^p_{(\zeta_i,\xi_i)})$. Let $\mathcal{M}_{\alpha_0} \in \mathfrak{F}$ such that $\mathcal{M}_{\alpha_0} \subset \bigcap_{i=1}^n \mathcal{M}_{\alpha(x^p_{(\zeta_i,\xi_i)})}$. Since $\mathcal{M}_{\alpha_0} \neq \tilde{0}$ there is some $1 \leq j \leq n$ such that $\mathcal{M}_{\alpha_0} \cap Cl(\mathcal{N}(x^p_{(\zeta_j,\xi_j)})) \neq \tilde{0}$. This implies that$\mathcal{M}_{\alpha(x^p_{(\zeta_j,\xi_j)})} \cap Cl(\mathcal{N}(x^p_{(\zeta_j,\xi_j)})) \neq \tilde{0}$ which is a contradiction .

(e) $\Rightarrow$ (d). Suppose $\exists$ a crisp $\mathcal{M} \in \mathcal{FFCS}(\mathbb{P})$ and a collection $\{\mathcal{F}_\alpha : \alpha \in \Lambda\}$ of $\mathcal{FFRC}$ sets of $\mathcal{P}$ such that each finite subcollection $\{\mathcal{F}_{\alpha_i} : i = 1, 2, 3, \ldots, n\}$ has a property that $(\cap_{i=1}^n Int(\mathcal{F}_{\alpha_i}))\cap\mathcal{M} \neq \tilde{0}$, but $(\cap_\alpha F_\alpha)\cap\mathcal{M} = \tilde{0}$. Then $(Int(\mathcal{F}_{\alpha_i}))\cap\mathcal{M}, \alpha \in \Lambda$, together with all finite intersection of the form $\cap_{i=1}^n (Int(\mathcal{F}_{\alpha_i})) \cap \mathcal{M}$, form a $\mathcal{FFFB}$ $\mathfrak{F}$ in $\mathcal{M}$. Then by (e) $\mathfrak{F}$ r-accumulates to some $\mathcal{FFP}$ $x^p_{(\zeta,\xi)} \in \mathcal{M}$. Thus $\forall$ $\mathcal{N}(x^p_{(\zeta,\xi)})$ containing $x^p_{(\zeta,\xi)}$ and each $Int(\mathcal{F}_\alpha)$, $Cl(\mathcal{N}(x^p_{(\zeta,\xi)})) \cap (Int(\mathcal{F}_\alpha) \cap \mathcal{M}) \neq \tilde{0}$. The fact $\mathcal{F}_\alpha \cap \mathcal{M} \neq \tilde{0}, \forall$ $\alpha \in \Lambda$ and the assumption that $(\cap_\alpha\mathcal{F}_\alpha) \cap \mathcal{M} = \tilde{0}$ give the existence of an $\alpha_0 \in \Lambda$ such that $x^p_{(\zeta,\xi)} \notin \mathcal{F}_{\alpha_0}$. Therefore , $x^p_{(\zeta,\xi)} \notin Int(\mathcal{F}_{\alpha_0})$ so that $x^p_{(\zeta,\xi)} \in (\mathcal{F}_{\alpha_0})^c \subset (Int(\mathcal{F}_{\alpha_0}))^c$. It then follows that $x^p_{(\zeta,\xi)} \in (\mathcal{F}_{\alpha_0})^c \subset Cl((\mathcal{F}_{\alpha_0})^c) \subset (Int(\mathcal{F}_{\alpha_0}))^c$

which implies $Cl((\mathcal{F}_{\alpha_0})^c) \cap Int(\mathcal{F}_{\alpha_0}) = \tilde{0}$, but this means $\mathfrak{F} \propto_r$ x. The contradiction gives $(\cap_\alpha F_\alpha) \cap \mathcal{M} \neq \tilde{0}$.

(e) $\Rightarrow$ (f) Let $\mathfrak{M} = \{\mathcal{M}_\alpha : \alpha \in \Lambda\}$ be a maximal $\mathcal{FFFB}$ in a crisp set $\mathcal{M} \in \mathcal{FFCS}(\mathbb{P})$. Then by (e) $\mathfrak{M} \underset{r}{\propto} x^p_{(\zeta,\xi)} \in \mathcal{M}$ so that $\mathfrak{M} \underset{r}{\rightarrow} x^p_{(\zeta,\xi)}$ by Theorem 3.5(c).

(f) $\Rightarrow$ (e) Let $\mathfrak{F} = \{\mathcal{M}_\alpha : \alpha \in \Lambda\}$ be a $\mathcal{FFFB}$ in a crisp set $\mathcal{M} \in \mathcal{FFCS}(\mathbb{P})$. Then $\exists$ a maximal $\mathcal{FFFB}$ $\mathfrak{M}$ such that $\mathfrak{M} \subset \mathfrak{F}$. By (f) $\mathfrak{M} \underset{r}{\rightarrow} x^p_{(\zeta,\xi)}$. Appling Theorem 3.5(a) and (b) we obtain that $\mathfrak{F} \underset{r}{\propto} x^p_{(\zeta,\xi)}$. □

Theorem 4.9. *In a $\mathcal{FFTS}$ $(\mathbb{P}, \Omega)$ the next statements are equivalent:*

(a) *$\mathbb{P}$ is $\mathcal{FFC}$-compact.*
(b) *For each crisp set $\mathcal{M} \in \mathcal{FFCS}(\mathbb{P})$ and each $\mathcal{FFN}$ ϕ in $\mathcal{M}$, $\exists$ a $\mathcal{FFP}$ $x^p_{(\zeta,\xi)} \in \mathcal{M}$ such that $\phi \underset{r}{\propto} x^p_{(\zeta,\xi)}$.*
(c) *For each crisp set $\mathcal{M} \in \mathcal{FFCS}(\mathbb{P})$ and each universal $\mathcal{FFN}$ ϕ in $\mathcal{M}$ $\exists$ a $\mathcal{FFP}$ $x^p_{(\zeta,\xi)} \in \mathcal{M}$ such that $\phi \underset{r}{\rightarrow} x^p_{(\zeta,\xi)}$.*

Proof. Obvious. □

Theorem 4.10. *Let $(\mathbb{P}, \Omega)$ be a $\mathcal{FFTS}$. Then the next conditions are equivalent:*

(a) *$\mathbb{P}$ is $\mathcal{FFC}$-compact.*
(b) *If $\mathcal{M}$ is a crisp$\mathcal{FFCS}$ set of $\mathbb{P}$ and $\mathfrak{F}$ is a collection of $\mathcal{FFRC}$ sets of $\mathbb{P}$ such that $\mathcal{M} \subseteq (\cap_{\mathcal{F}\in\mathfrak{F}}\mathcal{F})^c$ $\exists$ a finite number of elements of $\mathfrak{F}$ say $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \dots \mathcal{F}_n$, such that $\mathcal{M} \subseteq (\bigcap_{i=1}^n (Int(\mathcal{F}_i)))^c$.*

Proof. (a) $\Rightarrow$ (b). Suppose that $\mathbb{P}$ is $\mathcal{FFC}$-compact. Let $\mathcal{M}$ is a crisp $\mathcal{FFC}$ set of $\mathbb{P}$ and $\mathfrak{F}$ is a collection of $\mathcal{FFRC}$ sets of $\mathbb{P}$ such that $\mathcal{M} \subseteq (\bigcap_{\mathcal{F}\in\mathfrak{F}} \mathcal{F})^c = \bigcup_{\mathcal{F}\in\mathfrak{F}}(\mathcal{F}^c)$. Clearly $\mathfrak{U} = \{\mathcal{F}^c : \mathcal{F} \in \mathfrak{F}\}$ is a $\mathcal{FFRO}$ cover of $\mathcal{M}$. Since $\mathbb{P}$ is $\mathcal{FFC}$-compact, by Theorem 4.8(b), $\mathfrak{U}$ has a finite subcover $\{\mathcal{F}_i^c : i = 1, 2, 3, \dots n\}$ such that $\mathcal{M} \subseteq \bigcup_{i=1}^n (Cl(\mathcal{F}_i^c))$. But, $\bigcup_{i=1}^n (Cl(\mathcal{F}_i^c)) = (\bigcap_{i=1}^n (Int(\mathcal{F}_i,)))^c$ Hence, $\mathcal{M} \subseteq (\bigcap_{i=1}^n (Int(\mathcal{F}_i,)))^c$.

(b) $\Rightarrow$ (a). Let $\mathcal{M}$ is a crisp $\mathcal{FFC}$ set of $\mathbb{P}$. Let $\mathfrak{F}$ be a collection of $\mathcal{FFRO}$ sets of $\mathbb{P}$ such that $\mathcal{M} \subseteq (\bigcup_{\mathcal{F}\in\mathfrak{F}} \mathcal{F})$. Put $\mathfrak{U} = \{\mathcal{F}^c : \mathcal{F} \in \mathfrak{F}\}$. Then $\mathfrak{U}$ is clearly a collection of $\mathcal{FFRC}$ sets of $\mathbb{P}$ such that $\mathcal{M} \subseteq \bigcup_{\mathcal{F}\in\mathfrak{F}} \mathcal{F} = \bigcup_{\mathcal{F}\in\mathfrak{F}} (\mathcal{F}^c)^c = (\bigcap_{\mathcal{F}\in\mathfrak{F}} \mathcal{F}^c)^c$. Hence by (b) $\exists$ a finite number of elements, say $\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \dots \mathcal{F}_n$, such that $\mathcal{M} \subseteq (\bigcap_{i=1}^n (Int(F_i^c)))^c = (\bigcup_{i=1}^n Cl(F_i)$. Hence, $\mathbb{P}$ is $\mathcal{FFC}$-compact. □

Definition 4.11. Let $\mathfrak{F}$ be a $\mathcal{FFFB}$. Then the $\mathcal{FFS}$ $\cap\{Cl(\mathcal{F}) : \mathcal{F} \in \mathfrak{F}\}$ is called Fermatean fuzzy adherent ($\mathcal{FFA}$) set of $\mathfrak{F}$.

Definition 4.12. A $\mathcal{FFFB}$ $\mathfrak{F}$ is said to be $\mathcal{FFA}$ convergent if every $\mathcal{FFO}$ neighborhood of the $\mathcal{FFA}$ set of $\mathfrak{F}$ contains an element of $\mathfrak{F}$.

Theorem 4.13. *A $\mathcal{FFTS}$ $(\mathbb{P}, \Omega)$ is $\mathcal{FFC}$-compact if and only if every $\mathcal{FFO}$ filter base is $\mathcal{FFA}$ convergent.*

Proof. Necessity: Let $(\mathbb{P},\Omega)$ be a $\mathcal{FFC}$-compact and $\mathfrak{F}$ be a $\mathcal{FFO}$ filter base with the $\mathcal{FFO}$ set $\mathcal{M}$. Let *mathcalN* $\in \Omega$ be a crisp set containing $\mathcal{M}$. Then the collection $\mathfrak{U} = \{(Cl(\mathcal{F}))^c : \mathcal{F} \in \mathfrak{F}\}$ be a $\mathcal{FFO}$ cover of crisp $\mathcal{FFC}$ set $\mathcal{N}^c$ of $\mathbb{P}$). Since $\mathbb{P}$ is $\mathcal{FFC}$-compact, $\exists$ a finite sub family $\{(Cl(\mathcal{F}_i))^c : i = 1,2,3\ldots n\}$ of $\mathfrak{U}$ such that $\mathcal{N}^c \subset \bigcup_{i=1}^n\{(Cl(\mathcal{F}_i))^c\} \subset \bigcup_{i=1}^n\{\mathcal{F}_i^c\} = \bigcap_{i=1}^n\{\mathcal{F}_i\}$. It follows that, $\bigcap_{i=1}^n\{\mathcal{F}_i\} \subset \mathcal{N}$. Since $\mathfrak{F}$ is a $\mathcal{FFFB}$ $\exists \mathcal{F} \in \mathfrak{F}$ such that $\mathcal{F} \subset \bigcap_{i=1}^n\{\mathcal{F}_i\} \subset \mathcal{N}$. Hence $\mathfrak{F}$ is $\mathcal{FFA}$ convergent.

Sufficiency: Assume that $\mathbb{P}$ is not $\mathcal{FFC}$-compact and every $\mathcal{FFO}$ filter base is $\mathcal{FFA}$ convergent. Therefore, $\exists$ a crisp $\mathcal{FFC}$ set $\mathcal{F}$ and a $\mathcal{FFO}$ cover $\mathfrak{U} = \{\mathcal{G}_\alpha\}_{\alpha\in\Lambda}$ of $\mathcal{F}$ such that $\mathcal{F} \not\subset \bigcup_{i=1}^n\{Cl(G_{\alpha i,})\}$ for every finite sub family of $\mathfrak{U}$. Let $\mathcal{V}_n = \{(Cl(G_{\alpha_i}))^c : i = 1,2,3\ldots n\}$. Then $\{V_n\}$ is a $\mathcal{FFO}$ filter base. Now, $\bigcap\{Cl(\mathcal{V}_n)\} = \bigcap_{i=1}^n\{(Cl(\mathcal{G}_{\alpha_i}))^c\} \subset \bigcap_{i=1}^n\{(\mathcal{G}_{\alpha_i})^c\} \subset \mathcal{F}^c$. Therefore $\exists$ $\mathcal{V}_n$ contained in $\mathcal{F}^c$. Hence, $\mathcal{F} \subset \bigcup_{i=1}^n\{(Cl(\mathcal{G}_{\alpha_i}))\}$, which is a contradiction. □

Theorem 4.14. *Let $\varphi : (\mathbb{P},\Omega) \to (\mathbb{Q},\tau)$ be a Fermatean fuzzy continuous surjective mapping and $\mathbb{P}$ is $\mathcal{FFC}$-compact. Then Q is $\mathcal{FFC}$-compact.*

Proof. Let $\mathcal{M}$, be a crisp $\mathcal{FFC}$ set of $\mathbb{Q}$. Let $\mathfrak{U} = \{\mathcal{G}_\alpha : \alpha \in \Lambda\}$ be a $\mathcal{FFO}$ cover of $\mathbb{Q}$. Since φ is Fermatean fuzzy continuous,$\varphi^{-1}(\mathcal{M})$ is a crisp $\mathcal{FFCS}$ of $\mathbb{P}$ and $\{\varphi^{-1}(\mathcal{G}_\alpha) : \alpha \in \Lambda\}$ is a $\mathcal{FFO}$ cover of $\varphi^{-1}(\mathcal{M})$ in $\mathbb{P}$. Since $\mathbb{P}$ is $\mathcal{FFC}$-compact, there exists a finite sub family $\{\varphi^{-1}(\mathcal{G}_{\alpha_1}), \varphi^{-1}(\mathcal{G}_{\alpha_2}), \varphi^{-1}(\mathcal{G}_{\alpha_{3,}})\ldots, \varphi^{-1}(\mathcal{G}_{\alpha_n})\}$ such that $\varphi^{-1}(\mathcal{M}) \subseteq \bigcup_{i=1}^n\{Cl(\varphi^{-1}(\mathcal{G}_{\alpha_i}))\} \subseteq \bigcup_{i=1}^n\{\varphi^{-1}(Cl(\mathcal{G}_{\alpha_i}))\}$. It follows that $\mathcal{M} \subseteq \bigcup_{i=1}^n\{Cl(\mathcal{G}_{\alpha_i})\}$. Hence, $\mathbb{Q}$ is $\mathcal{FFC}$-compact. □

Definition 4.15. Let $(\mathbb{P},\Omega)$ be a $\mathcal{FFTS}$ and $\mathbb{Y}$ be a nonempty crisp subset of $\mathbb{P}$. Then $\Omega_{\mathbb{Y}} = \{\mathcal{M} \cap \mathbb{Y} : \mathcal{M} \in \Omega\}$, is said to be the Fermatean fuzzy relative topology on $\mathbb{Y}$ and $(\mathbb{Y},\Omega_{\mathbb{Y}})$ is called a Fermatean fuzzy subspace ($\mathcal{FFSS}$) of $(\mathbb{P},\Omega)$.

Theorem 4.16. *Let $(\mathbb{Y},\Omega_{\mathbb{Y}})$ be a $\mathcal{FFSS}$ of a $\mathcal{FFTS}$ $(\mathbb{P},\Omega)$ and $\mathcal{M} \in FFS(\mathbb{P})$, then:*

(a) $\mathcal{M} \in \Omega_{\mathbb{Y}} \Leftrightarrow \mathcal{M} = \mathbb{Y} \Cap \mathcal{O}$ *for some* $\mathcal{O} \in \Omega$.

(b) $\mathcal{M} \in \mathcal{FFCS}(\mathbb{Y}) \Leftrightarrow \mathcal{M} = \mathbb{Y} \Cap \mathcal{F}$ *for some* $\mathcal{F} \in FFSC(\mathbb{P})$.

Theorem 4.17. *Let $(\mathbb{Y},\Omega_{\mathbb{Y}})$ be a $\mathcal{FFSS}$ of a $\mathcal{FFTS}$ $(\mathbb{P},\Omega)$ and$\mathcal{M} \in \Gamma_{\mathbb{Y}}$. If $\mathbb{Y} \in \Omega$ then $\mathcal{M} \in \Omega$.*

Theorem 4.18. *Let $(\mathbb{Y},\Omega_{\mathbb{Y}})$ be a $\mathcal{FFSS}$ of a $\mathcal{FFTS}$ $(\mathbb{P},\Omega)$. Then a $\mathcal{FFS}$ $\mathcal{M}_{\mathbb{Y}} \in FFCS(\mathbb{Y}) \Rightarrow \mathcal{M}_{\mathbb{Y}} \in FFCS(\mathbb{P}) \Leftrightarrow \mathbb{Y} \in FFCS(\mathbb{P})$.*

Definition 4.19. A crisp subset $\mathcal{M}$ of a $\mathcal{FFTS}$ $(\mathbb{P},\Omega)$ is called $\mathcal{FFC}$-compact if the $\mathcal{FFSS}$ $(\mathbb{M},\Omega_M)$ is $\mathcal{FFC}$-compact.

Definition 4.20. A subset $\mathcal{M}$ of a $\mathcal{FFTS}$ $(\mathbb{P},\Omega)$ is called $\mathcal{FFC}$-compact relative to Ω if every $\mathcal{FFO}$ cover of $\mathcal{M}$ has a finite subfamily whose closure covers $\mathcal{M}$.

Theorem 4.21. *Every Fermatean fuzzy closed open crisp subset of a $\mathcal{FFC}$-compact space is $\mathcal{FFC}$-compact.*

Theorem 4.22. *Every Fermatean fuzzy closed crisp subset $\mathcal{M}$ of an $\mathcal{FFC}$-compact space $(\mathbb{P},\Omega)$ is $\mathcal{FFC}$-compact relative to Ω .*

Proof. Follows from Definition4.20 and Theorem 4.18 □

Theorem 4.23. *A $\mathcal{FFTS}$ $(\mathbb{P}, \Omega)$ is $\mathcal{FFC}$-compact if $\mathbb{P}$ is the finite union of $\mathcal{FFO}$ C-compact crisp subsets.*

Proof. suppose $\mathbb{P} = \mathcal{M} \cup \mathcal{N}$ where $\mathcal{M}$ and $\mathcal{N}$ are $\mathcal{FFO}$ crisp subsets of $\mathbb{P}$ and $(\mathcal{M}, \Omega_{\mathcal{M}}), (\mathcal{N}, \Omega_{N})$ are $\mathcal{FFC}$-compact. Let $\mathcal{K}$ be a crisp $\mathcal{FFCS}$ in $\mathbb{P}$ and $\{\mathcal{G}_\alpha : \alpha \in \Lambda\}$ be a $\mathcal{FFO}$ cover of $\mathcal{K}$. Since $\mathcal{M} \in \Omega$, $\{\mathcal{G}_\alpha \cap \mathcal{M} : \alpha \in \Lambda\}$ is a $\Omega_{\mathcal{M}} - \mathcal{FFO}$ cover of the $\Omega_{\mathcal{M}} - \mathcal{FFC}$ crisp subset $\mathcal{K} \cap \mathcal{M}$ of $\mathcal{M}$. By hypothesis

$\mathcal{K} \cap \mathcal{M} \subset \bigcup_{i=1}^{n} \Omega_{\mathcal{M}} - Cl(\mathcal{G}_{\alpha_i}) \cap \mathcal{M} \subset \bigcup_{i=1}^{n} Cl(\mathcal{G}_{\alpha_i})$ for some $n \in \mathbf{N}$.

Similarly

$K \cap \mathcal{N} \subset \bigcup_{j=1}^{m} Cl(\mathcal{G}_{\beta_j})$, for some $m \in \mathbf{N}$.

Hence, $\mathcal{K} \subset (\bigcup_{i=1}^{n} Cl(\mathcal{G}_{\alpha_i})) \bigcup (\cup_{j=1}^{m} Cl(\mathcal{G}_{\beta_j}))$ which implies that $\mathbb{P}$ is $\mathcal{FFC}$ -compact. □

Theorem 4.24. *A $\mathcal{FFTS}$ $(\mathbb{P}, \Omega)$ is $\mathcal{FFC}$-compact if $\mathbb{P}$ is the finite union of subsets of $\mathbb{P}$ which are $\mathcal{FFC}$-compact relative to Ω.*

Proof. Similar to that of Theorem 4.23 □

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