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ESTIMATES OF LOGARITHMIC COEFFICIENTS AND INVERSE LOGARITHMIC COEFFICIENTS ON A SUBCLASS OF ANALYTIC UNIVALENT FUNCTIONS

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Abstract

Consider the class $\mathcal{S}$ of analytic and univalent functions defined in the unit disk $|\zeta| < 1$ normalized by the expansion $h(\zeta) = \zeta + \sum_{k\geq 2} b_k\zeta^k$. For each function $h \in \mathcal{S}$ its inverse $H = h^{-1}$ can be represented as $H(\xi) = \xi + \sum_{k\geq 2} B_k\xi^k$ valid for $|\xi| < \frac{1}{4}$. The logarithmic coefficients and inverse logarithmic coefficients are given by $\log(h(\zeta)/\zeta) = 2\sum_{k\geq 1}\lambda_k(h)\zeta^k$ and $\log(h(\xi)/\xi) = 2\sum_{k\geq 1}\Lambda_k(h)\xi^k$ respectively. In this research article we compute the upper bounds for the logarithmic coefficients λ_k and inverse logarithmic coefficients $\Lambda_k, k = 1, 2, 3$ of two subclasses of analytic functions namely $S_D(\eta, \upsilon)$ and $P^{\tau}_{\delta}(\upsilon)$

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1. Introduction

Let $\mathcal{A}$ be the class of all analytic function of the form $h(\zeta) = \zeta + \sum_{k\geq 2} b_k\zeta^k$ defined in an open unit disk $\mathcal{U} = \{\zeta : |\zeta| < 1\}$ with normalization conditions $h(0) = 0$ and $h'(0) = 1$. A function belonging to $\mathcal{A}$ is referred to as univalent if it is injective on $\mathcal{U}$. The subclass of all such univalent functions is represented by $\mathcal{S}$.
Let $\mathcal{P}$ be the class of all functions analytic in $\mathcal{U}$ for which $\Re\{p(\zeta)\} > 0$ and

$$p(\zeta) = 1 + c_1\zeta + c_2\zeta^2 + \ldots, \quad \text{for} \quad \zeta \in \mathcal{U}. \tag{1.1}$$

For $h \in \mathcal{S}$, the logarithmic coefficients λ_k [6] are defined by the expansion

$$\log\left(\frac{h(\zeta)}{\zeta}\right) = 2\sum_{k\geq 1}\lambda_k\zeta^k. \tag{1.2}$$

The investigation of logarithmic coefficients has significantly contributed to the development of the theory of univalent functions. In the 1960's Milin [9] conducted pioneering work by examining how the properties of logarithmic coefficients could be transferred to the Taylor coefficients of univalent functions and their powers. His conjectured inequalities became important to this field as their validity implied several important results including the Robertson and Bieberbach conjectures as seen in

[4, 9, 12]. These inequalities attracted considerable attention due to their far-reaching implications in geometric function theory. In a major breakthrough Louis de Branges [1] proved Milin's conjectures in 1984. Specifically, it established that for any function $h \in \mathcal{S}$ the inequality $|b_k| \leq k$ holds with equality if and only if h is a rotation of the Koebe function $k(\zeta) = \frac{\zeta}{(1-\zeta)^2}$. The sharp upperbounds for λ_k has been established for subclasses with Bazilevič function [2], close to convex function [3] and univalent functions [5]. For the subclass of starlike univalent functions the inequality $|\lambda_k| \leq \frac{1}{k}, k \geq 1$ is true but fails in the order of magnitude for the entire subclass $\mathcal{S}$. Ye [15] and Murat Cagler [10] determined the estimates of the general logarithmic coefficient for close to convex function.

The inverse of a function $h \in \mathcal{S}$ denoted by $H = h^{-1}$ admits the Taylor series expansion

$$H(\xi) = \xi + \sum_{k\geq 2} B_k \xi^k$$

which holds for $|\xi| < \frac{1}{4}$ in accordance with Koebe's one-quarter theorem. A significant result concerning these inverse coefficients was obtained by Lowner (cf. [6]) who employed a variational method to establish the sharp bound $|B_k| \leq K_k$ where $K_k = \frac{(2k)!}{k!(k-1)!}$. These coefficients K_k appear in the Keobe function's inverse given by $K(\xi) = \xi + K_2\xi^2 + K_3\xi^3 + \ldots$. The problem of estimating the coefficients B_k has attracted substantial attention particularly when the function h belongs to specific geometric subclasses of $\mathcal{S}$. While many researchers have revisited this inequality offering various methods of proof a notably simplified and elegant proof was later contributed by Yang [16].

The initial logarithmic coefficients $\lambda_1, \lambda_2, \lambda_3$ for $h \in \mathcal{S}$ was obtained in the following manner

$$2\sum_{k\geq 1} \lambda_k b_k = b_2\zeta + b_3\zeta^2 + b_4\zeta^3 + \cdots - \frac{1}{2}(b_2\zeta + b_3\zeta^2 + b_4\zeta^3 + \cdots)^2 + \frac{1}{3}(b_2\zeta + b_3\zeta^2 + b_4\zeta^3 + \cdots)^3 + \cdots, \zeta \in \mathcal{U}.$$

Equating the coefficients of ζ^k for $k = 1, 2, 3$

$$2\lambda_1 = b_2 \tag{1.3}$$

$$2\lambda_2 = b_3 - \frac{1}{2}{b_2}^2 \tag{1.4}$$

$$2\lambda_3 = b_4 - b_2b_3 + \frac{1}{3}{b_2}^3. \tag{1.5}$$

A function $h \in \mathcal{S}$ is said to be subordinate to another function $g \in \mathcal{S}$ written as $h \prec g$ if there exists an analytic function ψ known as a Schwarz function defined in the unit disk $\mathcal{U}$ such that $\psi(0) = 0$ and $|\psi(\zeta)| < 1$ for all $\zeta \in \mathcal{U}$ and $h(\zeta) = g(\psi(\zeta))$.

In particular if the function g is univalent in $\mathcal{U}$ the relation $h \prec g$ holds if and only if

$$h(0) = g(0) \quad \text{and} \quad h(\mathcal{U}) \prec g(\mathcal{U}).$$

Ma and Minda [8] introduced the class $C(\phi)$ and $\mathcal{S}^*(\phi)$ of convex and starlike functions in which $\frac{\zeta h'(\zeta)}{h(\zeta)}$ or $1+\frac{\zeta h''(\zeta)}{h'(\zeta)}$ are subordinate to a more naturally occurring superordinate function using the subordination principle. They took into consideration the analytic univalent function ϕ such that $\phi(0) = 1$ and $\phi'(0) > 0$ with the positive real part in $\mathcal{U}$, $\phi(\mathcal{U})$ is symmetric with respect to the real axis. The power series expansion of ϕ is $\phi(\zeta) = 1 + \sum_{k\geq 1} d_k\zeta^k$.
Symbolically,

$$\mathcal{S}^*(\phi) = \left\{h \in \mathcal{S} : \frac{\zeta h'(\zeta)}{h(\zeta)} \prec \phi(\zeta)\right\},$$
$$C(\phi) = \left\{h \in \mathcal{S} : 1 + \frac{\zeta h''(\zeta)}{h'(\zeta)} \prec \phi(\zeta)\right\}$$

In Geometric function theory many special cases of $\mathcal{S}^*(\phi)$ and $C(\phi)$ play an important role because of its geometric properties.
In this research article, we determine the upper bounds of the logarithmic coefficients $\lambda_1, \lambda_2, \lambda_3$ and the inverse logarithmic coefficients $\Lambda_1, \Lambda_2, \Lambda_3$ of functions in $S_D(\eta, \upsilon)$ and $P^\tau_\delta(\upsilon)$.

Definition 1.1 (cf. [13]). Let $h \in \mathcal{S}$ and $0 \leq \eta \leq 1, 0 \leq \upsilon < 1$ we say that $h \in S_D(\eta, \upsilon)$ if

$$\frac{\eta\left|h'(\zeta) - \frac{h(\zeta)}{\zeta}\right|}{\left|\frac{h(\zeta)}{\zeta} - \upsilon\right|} \leq 1, \zeta \in \mathcal{U}. \tag{1.6}$$

Definition 1.2 (cf. [14]). For $\tau \in C\backslash\{0\}$, we say that $h \in P^\tau_\delta(\upsilon)$ with $0 \leq \delta < 1$ if it satisfies the inequality

$$\left|\frac{(1-\delta)\frac{h(\zeta)}{\zeta} + \delta h''(\zeta) - 1}{2\tau(1-\upsilon) + (1-\delta)\frac{h(\zeta)}{\zeta} + \delta h''(\zeta) - 1}\right| < 1.$$

To establish our main results we utilize the following lemma

Lemma 1.3. *[7, 12] If $p(\zeta) \in \mathcal{P}$ and has the series of the form (1.1) , then*

$$|c_{n+k} - \mu c_n c_k| \leq 2$$
$$|c_n| \leq 2 \, for \, n \geq 1$$
$$|c_2 - \eta c_1^2| \leq 2\max\{1, |2\eta - 1|\}, \eta \in \mathbb{C}$$
$$|Mc_1^3 - Nc_1c_2 + Oc_3| \leq 2|M| + |N - 2M| + 2|M - N + O|.$$

2. Main Results

2.1. Logarithmic coefficients of Subclasses $S_D(\eta, \upsilon)$ and $P^\tau_\delta(\upsilon)$

THEOREM 2.1. *The logarithmic coefficients λ_k, $k = 1,2,3$ of $h \in S_D(\eta,\upsilon)$ satisfy the inequalities*

$$|\lambda_1| \leq \frac{|1-\upsilon|}{2|\eta|}$$
$$|\lambda_2| \leq \frac{|1-\upsilon|}{4|\eta|}\max\{1, \frac{|\upsilon|}{|\eta|}\}$$
$$|\lambda_3| \leq |1-\upsilon|\Big(\Big|\frac{(1-\upsilon)^2}{4\eta^3}\Big| + \Big|\frac{3(1-\upsilon)(1-\eta)}{8\eta^3}\Big| + \Big|\frac{3(1-\eta)}{16\eta^2}\Big| + \Big|\frac{(1-\upsilon)}{8\eta^2}\Big| + \Big|\frac{9}{16\eta}\Big| + \Big|\frac{5}{8}\Big|\Big)$$

PROOF. Since $h \in S_D(\eta,\upsilon)$, $\exists$ a schwarz function ψ such that

$$\eta\Big(h'(\zeta) - \frac{h(\zeta)}{\zeta}\Big) = \psi(\zeta)\Big(\frac{h(\zeta)}{\zeta} - \upsilon\Big).$$

Equating the coefficients we have

$$b_2 = \frac{c_1(1-\upsilon)}{2\eta} \tag{2.1}$$
$$b_3 = \frac{(1-\upsilon)}{4\eta}(c_2 - \frac{(1-\eta)}{2\eta}c_1^2) \tag{2.2}$$
$$b_4 = (1-\upsilon)\Big(\frac{c_3}{3} + \Big(\frac{3}{8\eta} - \frac{1}{2}\Big)c_1c_2 + \Big(\frac{(1-\eta)}{16\eta^2} + \frac{1}{8}\Big)c_1^3\Big). \tag{2.3}$$

We know that

$$\psi(\zeta) = \frac{p(\zeta)-1}{p(\zeta)+1}$$

where $p(\zeta) \in \mathcal{P}$ given by (1.1) and by using Lemma 1.3 we get

$$|\lambda_1| \leq \frac{|1-\upsilon|}{2|\eta|}$$
$$|\lambda_2| \leq \frac{|1-\upsilon|}{4|\eta|}\max\{1, \frac{|\upsilon|}{|\eta|}\}$$
$$|\lambda_3| \leq |1-\upsilon|\Big(\Big|\frac{(1-\upsilon)^2}{4\eta^3}\Big| + \Big|\frac{3(1-\upsilon)(1-\eta)}{8\eta^3}\Big| + \Big|\frac{3(1-\eta)}{16\eta^2}\Big| + \Big|\frac{(1-\upsilon)}{8\eta^2}\Big| + \Big|\frac{9}{16\eta}\Big| + \Big|\frac{5}{8}\Big|\Big).$$

□

THEOREM 2.2. *The logarithmic coefficients λ_k, $k = 1,2,3$ of $h \in P_\delta^\tau(\upsilon)$ satisfy the inequalities*

$$|\lambda_1| \leq \frac{|(1-\upsilon)\tau|}{|1+\delta|}$$
$$|\lambda_2| \leq \frac{|(1-\upsilon)\tau|}{|1+2\delta|}\max\Big(1, \Big|\frac{\tau(1-\upsilon)(1+2\delta)}{(1+\delta)^2}\Big|\Big)$$
$$|\lambda_3| \leq |\tau(1-\upsilon)|\Big(\Big|\frac{2}{3(1+3\delta)}\Big| + \Big|\frac{3\tau(1-\upsilon)}{(1+\delta)(1+2\delta)}\Big| + \Big|\frac{\tau^2(1-\upsilon)^2}{(1+\delta)^3}\Big|\Big).$$

Proof. Since $h \in P_{\delta}^{\tau}(\upsilon)$, $\exists$ a Schwarz function ψ such that

$$(1-\delta)\frac{h(\zeta)}{\zeta} + \delta h''(\zeta) - 1 = \psi(\zeta)(2\tau(1-\upsilon) + (1-\delta)\frac{h(\zeta)}{\zeta} + \delta h''(\zeta) - 1).$$

Equating the coefficients we have

$$b_2 = \frac{(1-\upsilon)\tau c_1}{(1+\delta)} \tag{2.4}$$

$$b_3 = \frac{(1-\upsilon)\tau c_2}{(1+2\delta)} \tag{2.5}$$

$$b_4 = \frac{(2(1-\upsilon)\tau c_3}{3(1+3\delta)}. \tag{2.6}$$

We know that

$$\psi(\zeta) = \frac{p(\zeta)-1}{p(\zeta)+1}$$

where $p(\zeta) \in \mathcal{P}$ given by (1.1) and by using Lemma 1.3 we get

$$|\lambda_1| \le \frac{|(1-\upsilon)\tau|}{|1+\delta|}$$

$$|\lambda_2| \le \frac{|(1-\upsilon)\tau|}{|1+2\delta|} \max\left(1, \left|\frac{\tau(1-\upsilon)(1+2\delta)}{(1+\delta)^2}\right|\right)$$

$$|\lambda_3| \le |\tau(1-\upsilon)|\left(\left|\frac{2}{3(1+3\delta)}\right| + \left|\frac{3\tau(1-\upsilon)}{(1+\delta)(1+2\delta)}\right| + \left|\frac{\tau^2(1-\upsilon)^2}{(1+\delta)^3}\right|\right).$$

□

2.2. Inverse logarithmic coefficients of Subclasses $S_D(\eta,\upsilon)$ and $P_{\delta}^{\tau}(\upsilon)$

Let H represent the inverse function of $h \in \mathcal{S}$ defined using the Taylor series expansion in the neighbourhood of the origin

$$H(\xi) = h^{-1}(\xi) = \xi + \sum_{k\ge 1} B_k \xi^k \tag{2.7}$$

where $|\xi| < \frac{1}{4}$. The inverse logarithmic coefficients $\Lambda_k, k \in \mathbb{N}$ of H is defined by the expansion

$$\log\frac{H(\xi)}{\xi} = 2\sum_{k\ge 1} \Lambda_k \xi^k. \tag{2.8}$$

We notice that $h(H(\xi)) = \xi$, $H(0) = 0 = h(0)$ and $H'(0) = 1 = h'(0)$ where $\zeta = H(\xi)$ and find $h'(\zeta)H'(\xi) = 1$.
Further differentiation yields

$$h''(\zeta)(H'(\xi))^2 + h'(\zeta)H''(\xi) = 0$$

$$h'''(\zeta)(H'(\xi))^3 + 3h''(\zeta)H'(\xi)H''(\xi) + h'(\zeta)H'''(\xi) = 0$$

$$h^{iv}(\xi)(H'(\zeta))^4 + 6h'''(\xi)(H'(\zeta))^2H''(\zeta) + h''(\xi)(3(H''(\zeta))^2 + 3h'(\xi)H'''(\zeta) + H'''(\zeta)) + h'(\xi)H^{iv}(\zeta) = 0.$$

Setting $\zeta = 0, \xi = 0$ we obtain

$$B_2 = -b_2 \tag{2.9}$$
$$B_3 = -b_3 + 2{b_2}^2 \tag{2.10}$$
$$B_4 = -b_4 + 5b_2b_3 - 5{b_2}^3. \tag{2.11}$$

Comparing the coefficients of (2.7) and (2.8) we get

$$2\Lambda_1 = B_2$$
$$2\Lambda_2 = B_3 - \frac{1}{2}{B_2}^2$$
$$2\Lambda_3 = B_4 - B_2B_3 + \frac{1}{3}{B_2}^3.$$

Using (2.9), (2.10) and (2.11) the inverse logarithmic coefficients are computed as in [11]

$$2\Lambda_1 = -b_2 \tag{2.12}$$
$$4\Lambda_2 = -2b_3 - 3{b_2}^2 \tag{2.13}$$
$$3\Lambda_3 = -3b_4 + 12b_2b_3 - 10{b_2}^3. \tag{2.14}$$

Theorem 2.3. *If H is the inverse of $h \in S_D(\eta, \upsilon)$ then*

$$|\Lambda_1| \leq \frac{|1-\upsilon|}{2|\eta|}$$
$$|\Lambda_2| \leq \frac{|1-\upsilon|}{4|\eta|} \max\{1, \left|\frac{\upsilon}{\eta}\right|)$$
$$|\Lambda_3| \leq |1-\upsilon|\left(1 + \left|\frac{9}{8\eta}\right| + \left|\frac{(1-\upsilon)}{2\eta^2}\right| + \left|\frac{(1-\eta)}{4\eta^2}\right| + \left|\frac{3(1-\eta)(1-\upsilon)}{\eta^3}\right| + \left|\frac{5(1-\upsilon)^2}{\eta^3}\right|\right).$$

Proof. We make use of the equations (2.1), (2.2) and (2.3) in (2.12), (2.13) and (2.14) to get the following expression

$$\Lambda_1 = \frac{-c_1(1-\upsilon)}{4\eta}$$
$$\Lambda_2 = \frac{-(1-\upsilon)}{8\eta}\left(c_2 - \frac{(\eta-\upsilon)}{2\eta}c_1^2\right)$$
$$\Lambda_3 = \frac{-(1-\upsilon)}{2}c_3 + \left(\frac{3(1-\upsilon)}{8\eta} - \frac{(1-\upsilon)}{2} - \frac{3(1-\upsilon)^2}{2\eta^2}\right)c_1c_2$$
$$- \left(\frac{-(1-\upsilon)(1-\eta)}{16\eta^2} - \frac{(1-\upsilon)}{8} - \frac{3(1-\eta)(1-\upsilon)^2}{4\eta^3} - \frac{10(1-\upsilon)^3}{8\eta^3}\right)c_1^3.$$

Applying lemma 1.3 we obtain

$$|\Lambda_1| \leq \frac{|1-\upsilon|}{2|\eta|}$$
$$|\Lambda_2| \leq \frac{|1-\upsilon|}{4|\eta|}\max\{1, |\frac{\upsilon}{\eta}|)$$
$$|\Lambda_3| \leq |1-\upsilon|\Big(1 + \Big|\frac{9}{8\eta}\Big| + \Big|\frac{(1-\upsilon)}{2\eta^2}\Big| + \Big|\frac{(1-\eta)}{4\eta^2}\Big| + \Big|\frac{3(1-\eta)(1-\upsilon)}{\eta^3}\Big| + \Big|\frac{5(1-\upsilon)^2}{\eta^3}\Big|\Big).$$

□

Theorem 2.4. *If H is the inverse of $h \in P^{\tau}_{\delta}(\upsilon)$ then*

$$|\Lambda_1| \leq \frac{|(1-\upsilon)\tau|}{|1+\delta|}$$
$$|\Lambda_2| \leq \frac{|\tau(1-\upsilon)|}{|1+2\delta|}\max\left\{1, \left|\frac{3\tau(1-\upsilon)(1+2\delta)}{(1+\delta)^2} - 1\right|\right\}$$
$$|\Lambda_3| \leq |\tau(1-\upsilon)|\left(\left|\frac{4}{3(1+3\delta)}\right| + \left|\frac{12\tau(1-\upsilon)}{(1+\delta)(1+2\delta)}\right| + \left|\frac{20\tau^2(1-\upsilon)^2}{(1+\delta)^3}\right|\right).$$

Proof. Using (2.4), (2.5) and (2.6) in (2.12), (2.13) and (2.14) yields

$$\Lambda_1 = \frac{-(1-\upsilon)\tau c_1}{2(1+\delta)}$$
$$\Lambda_2 = \frac{-2\tau(1-\upsilon)|}{4(1+2\delta)}\left(c_2 - \frac{3\tau(1-\upsilon)(1+2\delta)}{2(1+\delta)^2}c_1^2\right)$$
$$\Lambda_3 = \frac{-2\tau(1-\upsilon)c_3}{3(1+3\delta)} + \frac{4\tau^2(1-\upsilon)^2}{(1+\delta)(1+2\delta)}c_1c_2 + \frac{10\tau^3(1-\upsilon)^3}{3(1+\delta)^3}c_1^3.$$

Applying lemma 1.3 we get

$$|\Lambda_1| \leq \frac{|(1-\upsilon)\tau|}{|1+\delta|}$$
$$|\Lambda_2| \leq \frac{|\tau(1-\upsilon)|}{|1+2\delta|}\max\left\{1, \left|\frac{3\tau(1-\upsilon)(1+2\delta)}{(1+\delta)^2} - 1\right|\right\}$$
$$|\Lambda_3| \leq |\tau(1-\upsilon)|\left(\left|\frac{4}{3(1+3\delta)}\right| + \left|\frac{12\tau(1-\upsilon)}{(1+\delta)(1+2\delta)}\right| + \left|\frac{20\tau^2(1-\upsilon)^2}{(1+\delta)^3}\right|\right).$$

□

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