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EXTENSIVE SUBCLASSES OF q-ANALOGUE OF ANALYTIC FUNCTIONS WITH SOME SUBORDINATION RESULTS

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Abstract

In present paper, we define new classes of starlike and convex analytic functions using the principle of q-calculus. The basic concepts of q-calculus that have been utilized in this investigation are also introduced. These classes are established using the well-known classes of convex and starlike functions. The investigation of these classes leads to the study of some geometrical notions, including coefficients estimates and other conclusions. Furthermore, some subordination results and integral representation for the functions of the class are investigated.

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1. Introduction

Quantum calculus is frequently used in mathematical sciences because of its numerous potential applications in number theory [25], combinatorics [10], orthogonal polynomials [11, 12, 28] and fundamental hypergeometric functions [9]. In [3, 5, 8], some of the fundamental ideas of q-calculus are demonstrated, along with how it is integrated into mathematical theories. In 1989, Srivastava published a book chapter that gave the right foundation for using the concepts of q-calculus within geometric function theory. In 1990, the q-calculus techniques were initially used in geometric function theory to define the notion of a q-starlike function. Some aspects of the use of quantum calculus in geometric function theory are highlighted in a recent paper [1] and Srivastava's review article [26] demonstrates additional advancements and a wide range of q-operators that are derived by using various operators that are unique to geometric function theory.

2. Preliminaries

Let A_p denote the class of all functions of the type:

$$f(z) = z^p + \sum_{k=1}^{\infty} a_{p+k} z^{p+k}, (p \in \mathbb{N} = 1, 2, 3...., z \in \mathbb{D}). \qquad (2.1)$$

such functions are analytic and multivalent in $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$. Let T_p denote the subclass of A_p, which consist of the functions f with the following power series expansion:

$$f(z) = z^p - \sum_{k=1}^{\infty} a_{p+k} z^{p+k}, (a_{p+k} > 0, p \in \mathbb{N} = 1, 2, 3...., z \in \mathbb{D}). \tag{2.2}$$

The class denoted by $S_p(\alpha)$ is called p-valent starlike functions of order α, it includes all functions $f \in A_p$ for which

$$\Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha, (0 \leq \alpha < p; \quad z \in \mathbb{D}), \tag{2.3}$$

where $\Re(f)$ denotes the real valued function. The class denoted by $K_p(\alpha)$ is called p-valent convex functions of order α, it includes all functions $f \in K_p$ for which

$$\Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha, (0 \leq \alpha < p; \quad z \in \mathbb{D}). \tag{2.4}$$

We note that: $S_p(0) = S_p, S_1(\alpha) = S(\alpha), K_p(0) = K_(p), K_1(\alpha) = K(\alpha)$ and

$$f(z) \in K_p(\alpha) \leftrightarrow (zf'(z))/p \in S_p(\alpha). \tag{2.5}$$

The concept of subordination initiated by Miller and Mocanu [6, 18, 19], considering that if f and $h \in A_p$, we say that f is subordinate to h, written as $f(z) \prec h(z)$, if there exist a Schwarz function $w \in A_p$ with $w(0) = 0$ and $|w(z)| < 1$ for all $z \in \mathbb{D}$ such that $f(z) = h(w(z)), z \in \mathbb{D}$. Furthermore, if the function $h \in A_p$, then the equivalency shown below is true: $f(z) \prec h(z) \leftrightarrow f(0) \prec h(0)$ and $f(\mathbb{D}) \prec h(\mathbb{D})$. (See [4, 6, 18, 19]).
Now we define a function g of class A_p by

$$g(z) = z^p + \sum_{k=1}^{\infty} b_{p+k} z^{p+k}, (b_{p+k} \geq 0) \tag{2.6}$$

The Hadamard product for the functions f and g is expressed by

$$(f * g)(z) = z^p + \sum_{k=1}^{\infty} a_{p+k} b_{p+k} z^{p+k} = (g * f)(z) \tag{2.7}$$

It is stated that the function h with $h(0) = 1$ belongs to the remarkable class of Janowski functions, which is represented by $P[A, B]$ provided that the following subordination is met:

$$h(z) \prec \frac{1 + Az}{1 + Bz}(-1 \leq B < A \leq 1).$$

The Class $P[A, B]$ was first presented and investigated by Janowski [15].
Aspects of quantum calculus are included in this study to continue a promising line of research as Jackson's q-integral and q-derivative [13, 14] are functions with a wide

range of uses in mathematics and connected domains. The fundamental ideas of q-calculus used in this study are then presented.
The q-difference operator $\partial_q : A_p \to A_p$ is defined by

$$\partial_q h(z) = \left\{ \begin{array}{ccc} \frac{h(qz)-h(z)}{(q-1)z} & , & z \neq 0 \\ h'(z) & , & z = 0 \end{array} \right\}$$

If $k \in \mathbb{N}$ and $z \in \mathbb{D}$, it is known that: $\partial_q^0 h(z) = h(z)$,

$$\partial_q^1 h(z) = \partial_q h(z), \partial_q^2 h(z) = \partial_q(\partial_q h(z)), \partial_q^m h(z) = \partial_q(\partial_q^{m-1} h(z)), m \in \mathbb{N}.$$

(see [16]).
If $h \in T_p$ and given by (2.2)

$$\partial_q^1 h(z) = \partial_q h(z) = [p]_q z^p - \sum_{k=1}^{\infty} [p+k]_q a_{p+k} z^{p+k}, z \neq 0$$

$$\partial_q^m h(z) = \prod_{i=1}^{m} [p-i+1]_q z^{p-m} - \sum_{k=1}^{\infty} \left\{ \prod_{i=1}^{m} [k-i+1]_q \right\} a_{p+k} z^{p+k-m} (m \in \mathbb{N}, z \neq 0)$$

where $[k]_q = \frac{1-q^k}{1-q} = 1 + \sum_{n=1}^{k-1} q^n$, $[0]_q = 0$,

$$[k]_q! = \left\{ \begin{array}{ccc} [k]_q [k-1]_q .....[2]_q [1]_q & , & k = 1, 2, 3, ... \\ 1 & , & k = 0 \end{array} \right.$$

The q-difference operator is guided by the following fundamental principles.(See [2]).

$$\partial_q(cf(z) \pm dh(z))) = c\partial_q f(z) \pm d\partial_q h(z)$$

$$\partial_q(f(z)h(z)) = f(qz)\partial_q(h(z)) + h(z)\partial_q(f(z))$$

$$h(qz)h(z) \neq 0$$

$$\partial_q(\log f(z)) = \frac{\ln q}{q-1} \frac{\partial_q(f(z))}{f(z)}$$

where $f, h \in A_p$, with c and d being real or complex number.
After studying a variety of fractional calculus operators, we find it convenient to limit our analysis to the definitions provided by Owa [20] and Owa and Srivastava [21].

Definition 2.1. The fractional integral of order λ is defined for the function $f(z)$ by

$$D_z^{-\lambda} f(z) = \frac{1}{\Gamma(\lambda)} \int_0^z \frac{f(t)}{(z-t)^{1-\lambda}} dt (\lambda > 0)$$

where $f(z)$ is analytic in a simply connected region containing the origin and multiplicity of $(z-t)^{\lambda-1}$ is removed by requiring $\log(z-t)$ to be real when $(z-t) > 0$.

Definition 2.2. The fractional derivative of order λ is defined for the function $f(z)$ by

$$D_z^{\lambda} f(z) = \frac{1}{\Gamma(1-\lambda)} \frac{d}{dz} \int_0^z \frac{f(t)}{(z-t)^{\lambda}} dt$$

where $0 \leq \lambda < 1$, $f(z)$ is analytic in a simply connected region containing the origin and multiplicity of $(z-t)^{-\lambda}$ is removed as in Definition 2.1.

A fractional differ-integral operator that is extended has been proposed and studied by numerous academics. For more information, check Patel and Mishra's work [22], as well as [17, 24, 25], and [29]. Two well-known authors discussed an extended fractional differ-integral operator in [22], $\Omega_z^{\lambda,p} f : A_p \to A_p$ for the function of the type:

$$\Omega_z^{(\lambda,p)} f(z) = z^p + \sum_{k=1}^{\infty} \frac{\Gamma(p+1-\lambda)\Gamma(p+k+1)}{\Gamma(p+1)\Gamma(p+k-\lambda+1)} a_{p+k} z^{p+k}, \lambda \in (-\infty, p+1) \subset \mathbb{R}; p \in \mathbb{N}.$$

$$\Omega_z^{(\lambda,p)} f(z) = z^p {}_2F_1(1, p+1; p+1-\lambda; z) * f(z)), z \in \mathbb{D}; \lambda \in (-\infty, p+1),$$

where ${}_2F_1$ is the Gaussian hypergeometric function defined by:

$${}_2F_1(a, b; c; z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k}{(c)_k (1)_k} z^k, (a, b, c \in \mathbb{C}, c \notin \mathbb{Z}_0 = \{0, -1, -2, ...\})$$

and $(d)_k$ is the Pochhammer symbol given in terms of the Gamma function by:

$$(d)_k = \begin{cases} 1 & , \quad (k = 0; d \in \mathbb{C} \setminus \{0\}) \\ d(d+1)...(d+k-1) & , \quad (k \in \mathbb{N}; d \in \mathbb{C}) \end{cases}$$

We note that ${}_2F_1$ represents an analytic function in $\mathbb{D}$ [27].
Following the idea of A.H. El-Qadeem and M.A. Mamon [7], we introduced generalized q-variance differ-integral operator for any integer λ given by
$\partial_q \Omega_z^{(\lambda,p)}; z \in \mathbb{D}; \lambda \in (-\infty, p+1)$. For $\partial_q \Omega_z^{(\lambda,p)} f \in T_p$, we have

$$\partial_q \Omega_z^{(\lambda,p)} f(z) = [p]_q z^p - \sum_{k=1}^{\infty} \frac{\Gamma(p+1-\lambda)\Gamma(p+k+1)}{\Gamma(p+1)\Gamma(p+k-\lambda+1)} [p+k]_q a_{p+k} z^{p+k}, \lambda \in (-\infty, p+1) \subset \mathbb{R}; z \in \mathbb{D}. \tag{2.8}$$

It is easily seen from (2.8) that

$$z(\partial_q \Omega_z^{(\lambda,p)} f)' = (p-\lambda)(\partial_q \Omega_z^{(\lambda+1,p)} f) + \lambda(\partial_q \Omega_z^{(\lambda,p)} f). \tag{2.9}$$

The following lemmas will be required for this paper:

Lemma 2.3. [19] *Let q be univalent in the unit disc $\mathbb{D}$. Let ϕ be analytic in a domain containing $q(\mathbb{D})$. If $\frac{z(q'(z))}{\phi(q(z))}$ is starlike, then*

$$\frac{z(\psi'(z))}{\phi(\psi(z))} \prec \frac{z(q'(z))}{\phi(q(z))}, (z \in \mathbb{D}),$$

then $\psi(z) \prec q(z)$ and q is the best dominant.

Lemma 2.4. [23] *Let p and q are analytic in $\mathbb{D}$, q is convex univalent, α,β and γ are complex and $\gamma \neq 0$. Further assume that*

$$\mathbb{R}\left\{\frac{\alpha}{\gamma} + \frac{2\beta}{\gamma}q(z) + (1 + \frac{z(q''(z))}{q'(z)}\right\} > 0.$$

If $p(z) = 1 + c_1 z + c_2 z^2 + ...$ is analytic in $\mathbb{D}$ and satisfies

$$\alpha p(z) + \beta p^2(z) + \gamma z p'(z) \prec \alpha q(z) + \beta q^2(z) + \gamma z q'(z),$$

then $p(z) \prec q(z)$ and q is the best dominant.

Definition 2.5. Let $h \in A_p$ we define a class as subordinations involving generalized q-variance differ-integral operator, defined by:

$$A^{\lambda}_{p,k}(A, B) = \left\{h \in A_p : \frac{\partial_q \Omega_z^{(\lambda,p)} f(z)}{[p]_q \Omega_z^{(\lambda,p)} f(z)} \prec \psi(z) = \frac{1 + Az}{1 + Bz}\right\}$$

we note that $T^{\lambda}_{p,k}(A, B) = T_p \cap A^{\lambda}_{p,k}(A, B)$ and $T^{\lambda}_{p,k}(\psi)) = T_p \cap A^{\lambda}_{p,k}(\psi)$.

3. Subordination Results

Theorem 3.1. *Let q be convex univalent, $\alpha \neq 0; 0 \leq \lambda < p - 1; p > 1$. Further assume that*

$$\mathbb{R}\left\{\frac{(p-\lambda)(1-\alpha)^{-1}}{\alpha} + 2(p-\lambda)q(z) + \left(1 + \frac{z(q''(z))}{q'(z)}\right)\right\} > 0. \tag{3.1}$$

If $f \in A_p$ satisfies

$$\frac{\partial_q \Omega_z^{(\lambda+1,p)} f(z)}{\partial_q \Omega_z^{(\lambda,p)} f(z)}\left\{1 - \alpha + \frac{\partial_q \Omega_z^{(\lambda+2,p)} f(z)}{\partial_q \Omega_z^{(\lambda+1,p)} f(z)}\right\} \prec \left(\frac{(p-\lambda)(1-\alpha)^{-1}}{p-\lambda-1}\right)q(z) + \frac{\alpha(p-\lambda)}{p-\lambda-1}q^2(z) + \frac{\alpha}{p-\lambda-1}zq'(z). \tag{3.2}$$

then

$$\frac{\partial_q \Omega_z^{(\lambda+1,p)} f(z)}{\partial_q \Omega_z^{(\lambda,p)} f(z)} \prec q(z) \tag{3.3}$$

and q is the best dominant.

Proof. Set

$$\Phi(z) = \frac{\partial_q \Omega_z^{(\lambda+1,p)} f(z)}{\partial_q \Omega_z^{(\lambda,p)} f(z)}, (z \in \mathbb{D}). \tag{3.4}$$

Then the function Φ is analytic in $\mathbb{D}$ and $\Phi(0) = 1$. Therefore, differentiating (3.3) logarithmically with respect to z and using the identity (2.9) in the resulting equation, we have

$$\frac{\partial_q \Omega_z^{(\lambda+2,p)} f(z)}{\partial_q \Omega_z^{(\lambda+1,p)} f(z)} = \frac{1}{p-\lambda-1}\left\{\frac{z\Phi'(z)}{\Phi(z)} + (p-\lambda)\Phi(z) - 1\right\}. \tag{3.5}$$

Therefore from (3.1), we have

$$\frac{\partial_q\Omega_z^{(\lambda+1,p)}f(z)}{\partial_q\Omega_z^{(\lambda,p)}f(z)}\left\{1-\alpha+\frac{\partial_q\Omega_z^{(\lambda+2,p)}f(z)}{\partial_q\Omega_z^{(\lambda+1,p)}f(z)}\right\}=\left(\frac{(p-\lambda)(1-\alpha)^{-1}}{p-\lambda-1}\right)\Phi(z)$$
$$+\frac{\alpha(p-\lambda)}{p-\lambda-1}\Phi^2(z)+\frac{\alpha}{p-\lambda-1}z\Phi'(z). \tag{3.6}$$

Using (3.5) in (3.2), we have

$$\left(\frac{(p-\lambda)(1-\alpha)^{-1}}{p-\lambda-1}\right)\Phi(z)+\frac{\alpha(p-\lambda)}{p-\lambda-1}\Phi^2(z)+\frac{\alpha}{p-\lambda-1}z\Phi'(z)\prec\left(\frac{(p-\lambda)(1-\alpha)^{-1}}{p-\lambda-1}\right)q(z)$$
$$+\frac{\alpha(p-\lambda)}{p-\lambda-1}q^2(z)+\frac{\alpha}{p-\lambda-1}zq'(z).$$

Hence the result now follows by using Lemma 2.2. □

Theorem 3.2. *Let q be univalent in $\mathbb{D}, q(0)=1.$. Let $\frac{z(q'(z))}{q(z)}$ be starlike univalent in $\mathbb{D}$; $0\le\lambda<p-1; p>1$. If $f\in A_p$ satisfies*

$$(p-\lambda-1)\frac{\partial_q\Omega_z^{(\lambda+2,p)}f(z)}{\partial_q\Omega_z^{(\lambda+1,p)}f(z)}-\alpha(p-\lambda)\frac{\partial_q\Omega_z^{(\lambda+1,p)}f(z)}{\partial_q\Omega_z^{(\lambda,p)}f(z)}\prec\frac{zq'(z)}{q(z)}+(1-\alpha)(1-\lambda)-1 \tag{3.7}$$

then $\frac{z^{\alpha-1}\partial_q\Omega_z^{(\lambda+1,p)}f(z)}{(\partial_q\Omega_z^{(\lambda,p)}f(z))^\alpha}\prec q(z)$ and q is the best dominant.

Proof. Set

$$\Phi(z)=\frac{z^{\alpha-1}\partial_q\Omega_z^{(\lambda+1,p)}f(z)}{(\partial_q\Omega_z^{(\lambda,p)}f(z))^\alpha},(z\in\mathbb{D}). \tag{3.8}$$

By a simple calculation from (3.7), we have

$$(p-\lambda-1)\frac{\partial_q\Omega_z^{(\lambda+2,p)}f(z)}{\partial_q\Omega_z^{(\lambda+1,p)}f(z)}-\alpha(p-\lambda)\frac{\partial_q\Omega_z^{(\lambda+1,p)}f(z)}{\partial_q\Omega_z^{(\lambda,p)}f(z)}=\frac{z\Phi'(z)}{\Phi(z)}+(1-\alpha)(1-\lambda)-1. \tag{3.9}$$

By using (3.8) in (3.6), we have

$$\frac{z\Phi'(z)}{\Phi(z)}\prec\frac{zq'(z)}{q(z)}$$

and the result follows by an application of Lemma 2.1. □

4. Coefficient Estimate

Theorem 4.1. *Suppose that $f\in A_p$ be of the form (1.1) and $-1\le B<0$. Then $f\in A_{p,k}^\lambda(A,B)$ if the following inequality satisfied.*

$$\sum_{k=1}^{\infty}\left\{(A-1)-(B-1)\frac{[p+k]_q}{[p]_q}\right\}D_{p,k}^\lambda a_{p+k}<A-B,$$

where

$$D_{p,k}^\lambda=\frac{\Gamma(p+1-\lambda)\Gamma(p+k+1)}{\Gamma(p+1)\Gamma(p+k-\lambda+1)}. \tag{4.1}$$

Proof. Suppose that $f \in A_p$ and if $f \in A^{\lambda}_{p,k}(A,B)$, then $\frac{\partial_q \Omega_z^{(\lambda,p)} f(z)}{[p]_q \Omega_z^{(\lambda,p)} f(z)} \prec \frac{1+Az}{1+Bz}$.
Equivalently, $f \in A^{\lambda}_{p,k}(A,B)$ if and only if

$$\left| \frac{\left(\frac{\partial_q \Omega_z^{(\lambda,p)} f(z)}{[p]_q \Omega_z^{(\lambda,p)} f(z)} - 1 \right)}{A - B\left(\frac{\partial_q \Omega_z^{(\lambda,p)} f(z)}{[p]_q \Omega_z^{(\lambda,p)} f(z)} \right)} \right| < 1.$$

$$\left| \frac{\sum_{k=1}^{\infty} \left\{ [p+k]_q - [p]_q \right\} D^{\lambda}_{p,k} a_{p+k} z^{p+k}}{(A-B)[p]_q z^p + \sum_{k=1}^{\infty} \left\{ A[p]_q - B[p+k]_q \right\} D^{\lambda}_{p,k} a_{p+k} z^{p+k}} \right|$$

$$\leq \frac{\sum_{k=1}^{\infty} \left\{ [p+k]_q - [p]_q \right\} D^{\lambda}_{p,k} a_{p+k}}{(A-B)[p]_q - \sum_{k=1}^{\infty} \left\{ A[p]_q - B[p+k]_q \right\} D^{\lambda}_{p,k} a_{p+k}} < 1,$$

using (4.1), the proof is completed. □

Theorem 4.2. *Suppose that $f \in T_p$ be of the form* (1.2) *and $-1 \leq B < 0$. Then $f \in T^{\lambda}_{p,k}(A,B)$ if the following inequality satisfied.*

$$\sum_{k=1}^{\infty} \left\{ (A-1) - (B-1)\frac{[p+k]_q}{[p]_q} \right\} D^{\lambda}_{p,k} a_{p+k} < A - B,$$

where

$$D^{\lambda}_{p,k} = \frac{\Gamma(p+1-\lambda)\Gamma(p+k+1)}{\Gamma(p+1)\Gamma(p+k-\lambda+1)}. \tag{4.2}$$

Proof. Suppose that $f \in T_p$ and if $f \in T^{\lambda}_{p,k}(A,B)$, then $\frac{\partial_q \Omega_z^{(\lambda,p)} f(z)}{[p]_q \Omega_z^{(\lambda,p)} f(z)} \prec \frac{1+Az}{1+Bz}$.
Equivalently, $f \in T^{\lambda}_{p,k}(A,B)$ if and only if

$$\left| \frac{\left(\frac{\partial_q \Omega_z^{(\lambda,p)} f(z)}{[p]_q \Omega_z^{(\lambda,p)} f(z)} - 1 \right)}{A - B\left(\frac{\partial_q \Omega_z^{(\lambda,p)} f(z)}{[p]_q \Omega_z^{(\lambda,p)} f(z)} \right)} \right| < 1.$$

$$\left| \frac{-\sum_{k=1}^{\infty} \left\{ [p+k]_q - [p]_q \right\} D^{\lambda}_{p,k} a_{p+k} z^{p+k}}{(A-B)[p]_q z^p - \sum_{k=1}^{\infty} \left\{ A[p]_q - B[p+k]_q \right\} D^{\lambda}_{p,k} a_{p+k} z^{p+k}} \right| < 1.$$

Since $\{\mathbb{R}(z)\} \leq |z|$, we obtain

$$\mathbb{R}\left\{ \frac{\sum_{k=1}^{\infty} \left\{ [p+k]_q - [p]_q \right\} D^{\lambda}_{p,k} a_{p+k} z^{p+k}}{(A-B)[p]_q z^p - \sum_{k=1}^{\infty} \left\{ A[p]_q - B[p+k]_q \right\} D^{\lambda}_{p,k} a_{p+k} z^{p+k}} \right\} < 1. \tag{4.3}$$

We now select z value along the real axis such that $\frac{\partial_q \Omega_z^{(\lambda,p)} f(z)}{[p]_q \Omega_z^{(\lambda,p)} f(z)}$ is real. Upon clearing the denominator in (4.3) and letting $z \to 1^-$, we obtain (4.2). □

Corollary 4.3. *Let $f \in T^{\lambda}_{p,k}(A,B)$ then*

$$a_{p+k} \leq \frac{(A-B)[p]_q}{\left\{(A-1)[p]_q - (B-1)[p+k]_q\right\} D^{\lambda}_{p,k}}.$$

The result is sharp for the function

$$f(z) = z^p - \frac{(A-B)[p]_q}{\left\{(A-1)[p]_q - (B-1)[p+k]_q\right\} D^{\lambda}_{p,k}} z^{p+k}.$$

5. Integral Representations

Theorem 5.1. *Suppose that $f \in A_p$ be of the form* (1.1) *be in the class $f \in A^{\lambda}_{p,k}(\psi)$ if and only if there exist a Schwartz function $w(z)$ such that*

$$\Omega_z^{(\lambda,p)} f(z) = exp\left(\frac{\ln q}{q-1}\int_0^z [p]_q \psi(w(z)) d_q t\right).$$

In particular, if let $f \in A^{\lambda}_{p,k}(A,B)$,

$$\Omega_z^{(\lambda,p)} f(z) = exp\left(\frac{\ln q}{q-1}[p]_q \int_0^z \frac{1+AL(t)}{1+BL(t)} d_q t\right)$$

where $|L(z)| < 1$.

Proof. Since $f \in A_p$ is supposed to be in the class $f \in A^{\lambda}_{p,k}(\psi)$,
equivalently

$$\frac{\partial_q \Omega_z^{(\lambda,p)} f(z)}{[p]_q \Omega_z^{(\lambda,p)} f(z)} \prec \psi(z)$$

$$\frac{\partial_q \Omega_z^{(\lambda,p)} f(z)}{[p]_q \Omega_z^{(\lambda,p)} f(z)} = [p]_q \psi((w(z))$$

after integration, we obtained

$$\Omega_z^{(\lambda,p)} f(z) = exp\left(\frac{\ln q}{q-1}\int_0^z [p]_q \psi(w(z)) d_q t\right).$$

Again, from the condition of the class $f \in A^{\lambda}_{p,k}(A,B)$,

$$\left|\frac{w(z)-1}{A-Bw(z)}\right| < 1$$

where

$$w(z) = \frac{\partial_q \Omega_z^{(\lambda,p)} f(z)}{[p]_q \Omega_z^{(\lambda,p)} f(z)}$$

$$\frac{w(z)-1}{A-Bw(z)} = L(z)$$

then $|L(z)| < 1$.
Finally, we have

$$\frac{\partial_q \Omega_z^{(\lambda,p)} f(z)}{[p]_q \Omega_z^{(\lambda,p)} f(z)} = [p]_q \frac{1 + AL(z)}{1 + BL(z)}$$

on integration, we obtained

$$\log \Omega_z^{(\lambda,p)} f(z) = \frac{\ln q}{q-1} [p]_q \int_0^z \frac{1 + AL(t)}{1 + BL(t)} d_q t,$$

therefore,
we get

$$\Omega_z^{(\lambda,p)} f(z) = exp\left(\frac{\ln q}{q-1} [p]_q \int_0^z \frac{1 + AL(t)}{1 + BL(t)} d_q t \right).$$

□

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