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HYBRID DIFFERENTIAL TRANSFORM–FINITE DIFFERENCE METHOD FOR TRANSIENT HEAT CONDUCTION WITH TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY

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Abstract

The study presents a hybrid solution technique through the integration framework of the Differential Transform Method (DTM) and the Finite Difference Method (FDM) for the analysis of one-dimensional transient heat conduction problems involving both constant and temperature-dependent thermal conductivity, in which the Differential Transform Method is employed in the time domain so that the governing partial differential equation is transformed into an algebraic iterative form while the Finite Difference Method is utilized in the spatial domain for discretization, thereby providing an efficient computational framework for the analysis of transient heat conduction problems, and the proposed hybrid technique effectively handles both the linear case ($\beta = 0$) and the nonlinear case ($\beta \neq 0$), whereas the numerical simulations are carried out and the obtained results are analyzed in comparison with the corresponding linear model, consequently demonstrating that the DTM–FDM hybrid approach provides accurate, reliable, and computationally efficient solutions for transient heat conduction problems arising in applied mathematical and engineering sciences.

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1. Introduction

The Differential Transform Method (DTM) is a semi-analytical numerical technique derived from the Taylor series expansion, which transforms differential equations into recursive algebraic relations. DTM is highly efficient for handling time-dependent problems because it avoids direct discretization in the temporal domain and provides rapid convergence for smooth solutions. However, for strongly nonlinear or large spatial-domain problems, DTM alone may suffer from reduced stability and increased computational complexity.

On the other hand, the Finite Difference Method (FDM) is a widely used numerical discretization technique for spatial derivatives. FDM is simple to implement and computationally efficient for boundary-value problems. Nevertheless, conventional

FDM generally requires smaller time steps and finer meshes to maintain numerical stability in transient nonlinear heat conduction problems.

Therefore, combining DTM in the time domain with FDM in the spatial domain enables the proposed hybrid DTM–FDM framework to exploit the advantages of both methods simultaneously. The hybridization reduces computational cost, improves convergence characteristics, and provides stable solutions for both linear and nonlinear transient heat conduction problems with temperature-dependent thermal conductivity.

Several numerical and semi-analytical techniques have been proposed in the literature for solving transient heat conduction problems. Among these approaches, the Differential Transform Method (DTM) has attracted considerable attention due to its rapid convergence and capability to transform differential equations into algebraic recurrence relations. Chu and Chen [1] employed the differential transform to discuss the behaviors of nonlinear heat conduction problems. They proposed a hybrid method of differential transform and finite difference approach to solve the transient responses of a nonlinear heat conduction problem. Different parameters of the equation and boundary conditions are considered to verify the feasibility of the proposed method. Simulation results are illustrated and discussed in comparison with the linear case, showing that the hybrid method can achieve good results.

Wadkar et al. [2] applied the hybrid differential transformation and finite difference method to solve the nonlinear transient heat conduction problem. Chu and Chen [3] used the hybrid DTM and finite difference method to analyze the nonlinear transient heat conduction problem. Maerefat et al. [4] applied the hybrid differential transform and finite difference method (HDTFD) to solve the 2D transient nonlinear straight annular fin equation. Chen and Liu [5] considered two-point boundary-value problems using the differential transformation method and proposed an iterative procedure for both linear and nonlinear cases. Peng and Chen [6] applied a hybrid numerical technique combining the differential transformation and finite difference method to investigate an annular fin with temperature-dependent thermal conductivity. Chen and Liu [7] used the differential transformation method to solve two-point boundary-value problems. Sarwe and Kulkarni [8] applied this method to determine heat transfer in annular fins. Arikoglu and Ozkol [9] demonstrated the efficiency of the Differential Transform Method (DTM) in solving differential equation boundary value problems by transforming them into algebraic recurrence relations. Yu and Chen [10] applied the hybrid method to solve the transient thermal stress distribution in a perfectly elastic isotropic annular fin.Despite these developments, most previous studies primarily focused on specific geometries, steady-state conditions, or simplified nonlinear formulations.

Research Gap

Although several researchers have employed DTM, FDM, and hybrid numerical approaches for nonlinear heat conduction problems, limited attention has been given to one-dimensional transient conduction problems involving temperature-dependent thermal conductivity within a unified hybrid framework. Most existing studies focus on specific geometries or steady-state conditions, while detailed recurrence formulations and computational implementation procedures are often omitted. Therefore, the present study develops a generalized hybrid DTM–FDM approach capable of efficiently solving both linear and nonlinear transient heat conduction problems with improved numerical stability and computational efficiency.

Novelty of the Present Work

The main contributions of this work are summarized as follows:

1. Development of a hybrid DTM–FDM framework for transient heat conduction with temperature-dependent thermal conductivity.
2. Unified treatment of both linear and nonlinear heat conduction problems.
3. Derivation of detailed recurrence relations for nonlinear conductivity terms.
4. Improved computational efficiency through combined temporal transformation and spatial discretization.
5. Numerical investigation of the influence of thermal conductivity variation parameter β on transient temperature evolution.

The hybrid approach of the differential transform method and finite difference method is used for the numerical investigation of linear and nonlinear transient heat conduction problems. In this paper, the DTM is applied in the time domain to transform the governing partial differential equation into an algebraic iterative form, while FDM is employed in the spatial domain for discretization. The method is applied to one-dimensional transient heat conduction problems for both linear cases ($\beta = 0$) and nonlinear cases ($\beta \neq 0$) where thermal conductivity depends on temperature. Numerical simulations demonstrate the efficiency and accuracy of the proposed technique compared to conventional numerical schemes.

The structure of this paper is as follows: Section 2 presents the mathematical formulation of the problem, including the governing equations, initial conditions, and boundary conditions for both linear and nonlinear cases. Section 3 describes the proposed hybrid Differential Transform–Finite Difference Method (DTM–FDM) in detail, outlining the transformation procedure and discretization strategy. Section 4 provides the numerical simulation results, along with a discussion of the temperature distribution and the influence of temperature-dependent thermal conductivity. Finally, Section 5 concludes the study and outlines directions for future research.

2. Differential Transformation Method

A brief outline of the Differential Transformation Method (DTM) is given in this section. The differential transformation method is applied mainly to solve initial value problems. Let $x(t)$ be an analytic function in domain D. The Taylor series expansion function of $x(t)$ is of the form,

$$x(t) = \sum_{n=0}^{\infty} \frac{(t-t_0)^n}{n!} \left. \frac{d^n x(t)}{dt^n} \right|_{t=t_0}. \tag{2.1}$$

When $t_0 = 0$ in Eq. (1), the resulting expansion is known as the Maclaurin expansion of $x(t)$ [12], and is expressed as:

$$x(t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} \left.\frac{d^n x(t)}{dt^n}\right|_{t=0} \quad \forall t \in D. \tag{2.2}$$

The differential transform or the Taylor transform of the function $x(t)$ is defined as,

$$X(n) = T[x(t)] = \frac{H^n}{n!} \left.\frac{d^n x(t)}{dt^n}\right|_{t=0}, \quad n = 0, 1, 2, \ldots \tag{2.3}$$

The function $X(n)$, also expressed as $T[x(t)]$, denotes the differential transform of the original function $x(t)$ with respect to the origin. By convention, lowercase letters are used for original functions, where the differential transforms are expressed using the respective uppercase notations. Here in this context, H is defined as the time interval. Furthermore, $X(n)$ is often referred to as the spectral representation of $x(t)$ in the spectrum domain [1]. The inverse differential transform, formulated using the Taylor series expansion, is expressed as:

$$x(t) = \sum_{n=0}^{\infty} \left(\frac{t}{H}\right)^n X(n). \tag{2.4}$$

In engineering applications, applying Eq. (4) allows the expression $x(t)$ to be expressed in the form of a finite Taylor series along with a residual term, as shown below:

$$x(t) = \sum_{n=0}^{k} \left(\frac{t}{H}\right)^n X(n) + R_{k+1}(t). \tag{2.5}$$

The Differential Transformation Method (DTM) offers an efficient and reliable technique for solving differential equations. Its fundamental concept lies in replacing calculus operations with algebraic iterations. To construct the iterative equations, it is essential to establish the interaction between the transformed forms of the original function $x(t)$ and the corresponding transforms of its derivatives. This relationship is derived from the definition of the differential transform in combination with the Taylor series expansion, and is represented as [3].

$$\tilde{D}X(n) = T\left[\frac{dx(t)}{dt}\right] = \frac{n+1}{H} X(n+1). \tag{2.6}$$

The derivative $x'(t)$ under the differential transform is represented by $\tilde{D}X(n)$. Furthermore, an essential property in the solution procedure is the transformation of a product of functions under the differential transform. For two functions $x(t)$ and $y(t)$, with differential transforms $X(n)$ and $Y(n)$ respectively, the transformation of their product is expressed as:

$$T[x(t)y(t)] = X(n) \otimes Y(n) = \sum_{l=0}^{k} X(l)\, Y(n-l). \tag{2.7}$$

In Eq. (2.7), the operator $\otimes$ denotes the summation of all product terms located on the RHS of the equation.

To obtain the recurrence formulation of the proposed hybrid DTM–FDM method, the differential transformation of the governing equation is performed term by term. The transformed expressions for the time derivative, spatial derivative, and nonlinear terms are obtained using the properties of the Differential Transform Method and the Cauchy product rule.

For the time derivative,

$$\frac{\partial \Theta(x,n)}{\partial t} = (n+1)\,\Theta(x,n+1),$$

For the second spatial derivative,

$$\tilde{D}\left[\frac{\partial^2 \Theta}{\partial x^2}\right] = \frac{\partial^2 \Theta(x,n)}{\partial x^2},$$

For the nonlinear product term,

$$\tilde{D}\left(\theta \cdot \frac{\partial^2 \theta}{\partial x^2}\right) = \sum_{l=0}^{n} \Theta(x,l)\,\frac{\partial^2 \Theta(x,n-l)}{\partial x^2}.$$

Similarly, the transformation of the nonlinear gradient term becomes

$$\tilde{D}\left(\frac{\partial \theta}{\partial x}\right)^2 = \sum_{l=0}^{n} \frac{\partial \Theta(x,l)}{\partial x} \cdot \frac{\partial \Theta(x,n-l)}{\partial x}.$$

Substituting these transformed expressions into the governing heat conduction equation yields the recurrence relation used in the hybrid DTM–FDM formulation.

Using the differential transformation method, a differential equation is reformulated as a recurrent algebraic relation expressed in the spectrum domain. The proposed iterative equation yields the differential transform $X(n)$ of the unknown function $x(t)$, while $x(t)$ itself is reconstructed through the inverse differential transform applied to $X(n)$, as specified in Eq. (4) or Eq. (5). To improve the rate of approaching the solution and enhance accuracy, the overall domain D is generally divided into sub-intervals. The procedure begins with solving the governing equation in the initial segment using the differential transformation method. The terminal values of this segment are then assigned as the starting values for the subsequent sub-interval, and the equation is re-solved using these updated values. This process is systematically re-evaluated across all sub-intervals until the complete solution over the domain D is obtained [11].

3. Mathematical Representation of the Problem

The fundamental equation for heat conduction:The present study formulates the one-dimensional transient heat conduction problem, which is given by:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial \theta}{\partial x}\right), \quad 0 \le x \le 1. \tag{3.1}$$

The governing one-dimensional transient heat conduction equation is adopted from classical heat transfer theory and Fourier's law of heat conduction [13, 14].

Here, $\theta = \theta(x, t)$ refers to the temperature of a point x and time t, and k is denoted by the thermal conductivity.

It is assumed in this study that the thermal conductivity k changes linearly with temperature θ, which can be expressed as:

$$k = k_0(1 + \beta\theta), \tag{3.2}$$

The temperature-dependent thermal conductivity model employed in the present study is widely used in nonlinear heat transfer analysis [14].

Here, the parameter k_0 denotes thermal conductivity at the specified reference temperature. β represents the temperature coefficient of thermal conductivity and $x \in [0, 1]$.

The governing equation has the following nonlinear form when Eq. (3.2) is substituted into Eq. (3.1):

$$\frac{\partial \theta}{\partial t} = k_0 \frac{\partial}{\partial x}\left((1 + \beta\theta)\frac{\partial \theta}{\partial x}\right), \quad 0 \le x \le 1 \tag{3.3}$$

The boundary conditions corresponding to the problem are defined as:

$$\theta(0, t) = 1 \quad \text{at } x = 0, \tag{3.4}$$

$$\theta(1, t) = 0 \quad \text{at } x = 1, \tag{3.5}$$

And the corresponding initial condition is expressed as:

$$\theta(x, 0) = 0 \quad \text{at } t = 0. \tag{3.6}$$

Equations (3.1) – (3.6) together represent the complete mathematical formulation of the problem.

4. Solution of the Problem

Initially, Eq. (3.3), which represents the governing heat conduction equation, is expanded to express the spatial derivative terms explicitly.

$$\frac{\partial \theta}{\partial t} = k_0 \frac{\partial}{\partial x}\left[\frac{\partial \theta}{\partial x} + \beta\theta \cdot \frac{\partial \theta}{\partial x}\right]$$

$$\frac{\partial \theta}{\partial t} = k_0 \left[\frac{\partial^2 \theta}{\partial x^2} + \beta \cdot \frac{\partial}{\partial x}\left(\theta \cdot \frac{\partial \theta}{\partial x}\right)\right]$$

$$\frac{\partial \theta}{\partial t} = k_0 \left[\frac{\partial^2 \theta}{\partial x^2} + \beta\left(\frac{\partial \theta}{\partial x} \cdot \frac{\partial \theta}{\partial x} + \theta\frac{\partial^2 \theta}{\partial x^2}\right)\right], \tag{4.1}$$

Equation (4.1) is linear when $\beta = 0$ (constant conductivity) and nonlinear when $\beta \ne 0$. The solution procedure begins by applying the differential transform to both sides of the governing equations, thereby converting them from the time domain into the spectral domain. Applying the differential transform to Equation (**??**) with respect to the time variable t, where the time interval is set as $H = 1$ during the transformation process.

We represent the temperature as a time-series at each spatial location. For a function $\theta(x, t)$, the differential transform in time is

$$\Theta(x, n) = \frac{1}{n!} \left. \frac{d^n \theta(x, t)}{dt^n} \right|_{t=0}, \quad n = 0, 1, 2, \ldots$$

The time derivative transforms as, for $\frac{\partial \theta}{\partial t}$ we get:

$$\frac{\partial \Theta(x, n)}{\partial t} = (n + 1)\,\Theta(x, n + 1). \tag{4.2}$$

Using the Differential Transform Method (DTM) in the time domain to Eq. (4.1), the spatial derivatives operate only on the spatial components of the transformed function. Consequently, the second spatial derivative $\frac{\partial^2 \theta}{\partial x^2}$ of $\theta(x, t)$ can be expressed as:

$$\tilde{D}\left[\frac{\partial^2 \Theta}{\partial x^2}\right] = \frac{\partial^2 \Theta(x, n)}{\partial x^2}, \tag{4.3}$$

Here, $\tilde{D}$ is used to represent the differential transform of a derivative.

When employing the Differential Transform Method (DTM) in relation to time, the term $\frac{\partial \theta}{\partial x} \cdot \frac{\partial \theta}{\partial x}$ is represented using the Cauchy product rule. Considering the DTM expansion,

$$\theta(x, t) = \sum_{n=0}^{\infty} \Theta(x, n)\, t^n.$$

the first spatial derivative becomes

$$\frac{\partial \theta}{\partial x} = \sum_{n=0}^{\infty} \frac{\partial \Theta(x, n)}{\partial x}\, t^n.$$

Therefore, the product $\frac{\partial \theta}{\partial x} \cdot \frac{\partial \theta}{\partial x}$ is expressed as:

$$\left(\frac{\partial \theta}{\partial x}\right)^2 = \sum_{n=0}^{\infty} \left(\sum_{l=0}^{n} \frac{\partial \Theta(x, l)}{\partial x} \cdot \frac{\partial \Theta(x, n - l)}{\partial x} \right) t^n,$$

$$\tilde{D}\left(\frac{\partial \theta}{\partial x}\right)^2 = \sum_{l=0}^{n} \frac{\partial \Theta(x, l)}{\partial x} \cdot \frac{\partial \Theta(x, n - l)}{\partial x}. \tag{4.4}$$

Here, $\tilde{D}$ is used to represent the differential transform of a derivative.

The nonlinear term $\theta \frac{\partial^2 \theta}{\partial x^2}$ under the differential transformation technique is transformed using the Cauchy product rule and is given by:

$$\theta \frac{\partial^2 \theta}{\partial x^2} = \sum_{n=0}^{\infty} \left(\sum_{l=0}^{n} \Theta(x, l)\, \frac{\partial^2 \Theta(x, n - l)}{\partial x^2} \right) t^n,$$

$$\tilde{D}\left(\theta \cdot \frac{\partial^2\theta}{\partial x^2}\right) = \sum_{l=0}^{n} \Theta(x,l)\,\frac{\partial^2\Theta(x,n-l)}{\partial x^2}. \tag{4.5}$$

Substituting Eqs. (4.2)–(4.5) into Eq. (4.1), the transformed equation becomes:

$$(n+1)\Theta(x,n) = k_0\left\{\frac{\partial^2\Theta(x,n)}{\partial x^2} + \beta\left[\sum_{l=0}^{n}\frac{\partial\Theta(x,n-l)}{\partial x}\cdot\frac{\partial\Theta(x,l)}{\partial x}\right] + \beta\left[\sum_{l=0}^{n}\Theta(x,l)\,\frac{\partial^2\Theta(x,n-l)}{\partial x^2}\right]\right\}. \tag{4.6}$$

Equation (4.6) represents the recurrence relation for the transformed components, where $\Theta(x,n)$ represents the differential transform of $\theta(x,t)$. The first component on the RHS corresponds to the linear part of the heat conduction equation, while the summations represent the nonlinear contributions.

The differential transformation is subsequently employed for the boundary conditions specified in Equations (3.4) and (3.5).

$$\begin{cases}\Theta(0,n) = 1 & \text{for } n = 0,\\ \Theta(0,n) = 0 & \text{for } n = 1,2,3,\ldots,\end{cases} \tag{4.7}$$

$$\Theta(1,n) = 0, \tag{4.8}$$

Similarly, the initial condition specified in Eq. (3.6) is subjected to the differential transform.

$$\Theta(x,0) = 0. \tag{4.9}$$

After applying the Differential Transformation Technique (DTM) in the time domain, Eq. (4.6) is discretized in space using the Finite Difference Method (FDM) [3]. The computational spatial interval $0 \le x \le 1$ is split into N equal subintervals of size Δx. For the spatial discretization, the first and second derivatives are approximated using second-order central difference formulas. Accordingly, the governing equation at each grid point can be written as:

$$\begin{aligned}(n+1)\Theta_i^{(n+1)} = k_0\Bigg[&\frac{1}{\Delta x^2}\left(\Theta_{i+1}^{n} - 2\Theta_i^{n} + \Theta_{i-1}^{n}\right)\\ &+\beta\sum_{l=0}^{n}\frac{\left(\Theta_{i+1}^{(n-l)} - \Theta_{i-1}^{(n-l)}\right)\left(\Theta_{i+1}^{l} - \Theta_{i-1}^{l}\right)}{4\Delta x^2}\\ &+\beta\sum_{l=0}^{n}\Theta_i^{l}\cdot\frac{\Theta_{i+1}^{(n-l)} - 2\Theta_i^{(n-l)} + \Theta_{i-1}^{(n-l)}}{\Delta x^2}\Bigg].\end{aligned} \tag{4.10}$$

Where Θ_i^n represents the differential transform of $\Theta(x,n)$ corresponding to the grid location x_i. The boundary conditions are discretized using the finite difference methods, as specified in Eqs. (4.7) and (4.8); the following discrete forms are obtained.

$$\begin{cases}\Theta_0(n) = 1 & \text{for } n = 0,\\ \Theta_0(n) = 0 & \text{for } n = 1,2,3,\ldots,\end{cases} \tag{4.11}$$

So $\Theta_1(n) \to \Theta_N(n)$.

That's why the boundary condition

$$\Theta(1, n) = 0,$$

is written equivalently as

$$\Theta_N(n) = 0. \tag{4.12}$$

In Eq. (4.12), N represents the number of divisions along the spatial domain, considered in the discretization process. Correspondingly, as defined in Eq. (**??**), the initial condition is reformulated in its finite-difference representation as:

Now we discretize space $x_i = i\Delta x$, $i = 0, 1, 2, 3, \ldots, N$.

So at each grid point, $\Theta(x_i, 0) = \Theta_i(0)$. But $\Theta(x, 0) = 0$.

that means,

$$\Theta_i(0) = 0, \quad i = 0, 1, 2, \ldots, N. \tag{4.13}$$

The recurrence relation obtained in Eq. (4.10) can be reformulated in its discrete form by employing second-order central difference techniques for approximating the first and second spatial derivatives. The resulting expression is given as:

$$\begin{aligned}\Theta_i^{(n+1)} = \frac{k_0}{(n+1)}\Bigg[& \frac{1}{\Delta x^2}\left(\Theta_{i+1}^n - 2\Theta_i^n + \Theta_{i-1}^n\right) \\ & + \frac{\beta}{4\Delta x^2}\left(\sum_{l=0}^{n}\Theta_{i+1}^{(n-l)}\Theta_{i+1}^{l} + \sum_{l=0}^{n}\Theta_{i-1}^{(n-l)}\Theta_{i-1}^{l} - 2\sum_{l=0}^{n}\Theta_{i+1}^{(n-l)}\Theta_{i-1}^{l}\right) \\ & + \frac{\beta}{\Delta x^2}\left(\sum_{l=0}^{n}\Theta_{i+1}^{(n-l)}\Theta_i^{l} + \sum_{l=0}^{n}\Theta_{i-1}^{(n-l)}\Theta_i^{l} - 2\sum_{l=0}^{n}\Theta_i^{l}\Theta_i^{(n-l)}\right)\Bigg].\end{aligned} \tag{4.14}$$

This equation establishes the iterative scheme corresponding to the Differential Transform Method (DTM), in which the terms of higher degree in the transform series are systematically evaluated using the prescribed initial and boundary conditions.

5. Simulation Results

The analysis begins with the application of the Differential Transform Method (DTM) to Eqs. (3.3) for the case of linearity, corresponding to $\beta = 0$. The analysis is carried out under the boundary and initial conditions specified in Eqs. (3.4)–(3.6). For simplicity and without loss of generality, the thermal conductivity parameter is normalized by setting $k_0 = 1$ in this and the later case. The proposed hybrid scheme is then employed with a truncation order of $K = 30$ in the differential transform series and a time increment of $\Delta t = 0.001$.

Case I: Linear Case ($\beta = 0$) The analysis of the linear case ($\beta = 0$) is considered under the boundary conditions $\theta(0,t) = 1$ and $\theta(1,t) = 0$ together with the initial condition $\theta(x,0) = 0$. This graph is a 3D surface plot showing how the distribution of temperature $\theta(x,t)$ changes over position x and time t for the linear case ($\beta = 0$) of the heat conduction problem.

As shown in Figure 1, at early time period, when t is nearly zero (when $t \approx 0$), the heat conduction changes are still minimal. This means that in a mathematical series solution, many terms are not needed. Just a few terms are enough to get an accurate result because the solution converges quickly and the error is small.

Case II: Nonlinear Case ($\beta \neq 0$) In the second case, different values of β are considered. Figure 2 illustrates the temperature evolution (or time response) at the mid-point $x = 0.5$ for various values of the nonlinear parameter β. The red solid line represents the linear condition ($\beta = 0$), while the blue dashed and green dash-dotted lines represent the nonlinear conditions with $\beta = 0.5$ and $\beta = 1$, respectively. The results indicate that the solution increases more rapidly with larger values of β.

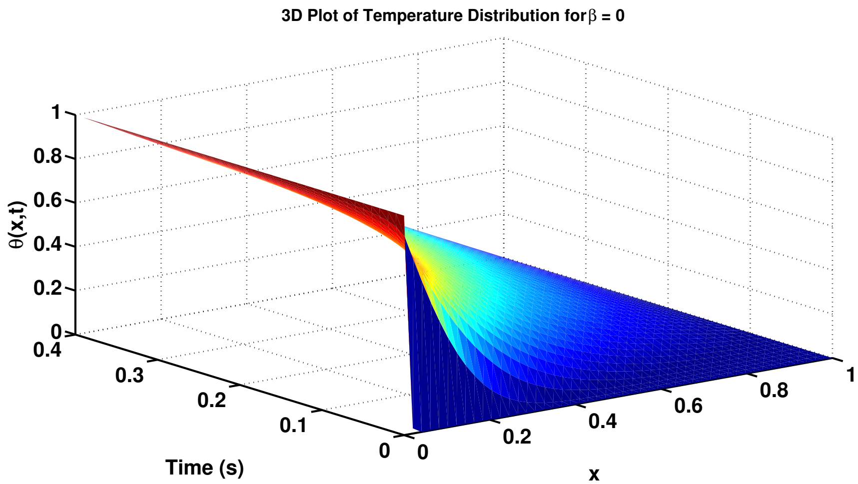
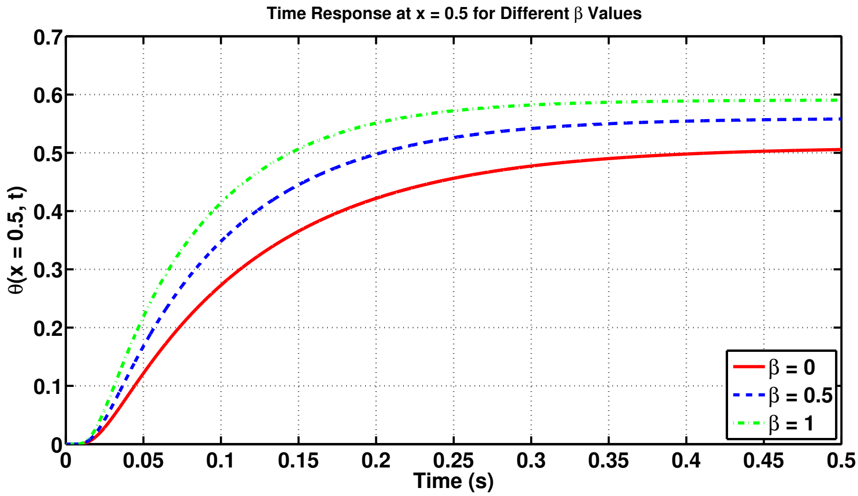


Figure 1. Three-dimensional temperature distribution $\theta(x,t)$ for the linear heat conduction case ($\beta = 0$) under the prescribed boundary and initial conditions.



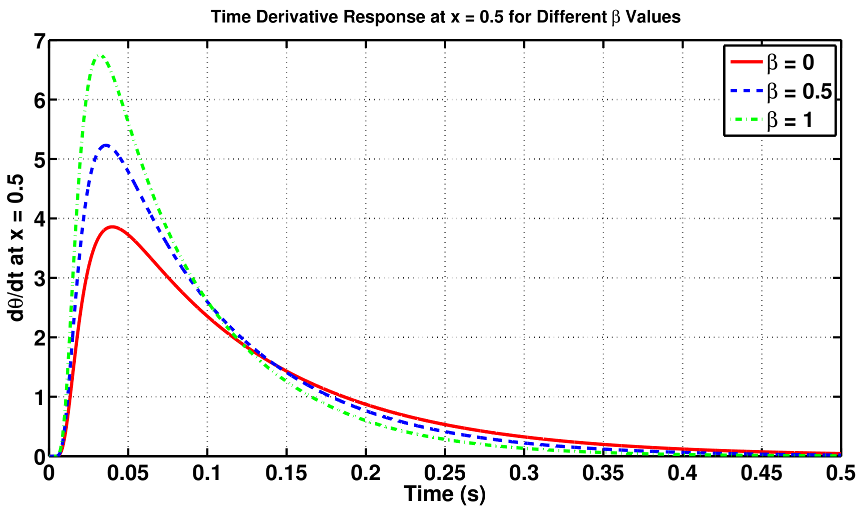


FIGURE 2. Time response of temperature $\theta(x = 0.5, t)$ and its corresponding time derivative $d\theta/dt$ at the same spatial location for different values of the nonlinear parameter β, under the prescribed boundary and initial conditions.

6. Discussion

It is observed that the DTM–FDM hybrid approach successfully resolves both linear and nonlinear formulations of the heat conduction equation. There are three governing parameters that significantly influence the accuracy of the solution. The first parameter is used in the finite difference discretization. The second governing parameter is the time interval Δt adopted in the differential transformation process. The third governing parameter is the degree of differential transform order n. A finer spatial discretization generally improves accuracy; however, using too many segments may lead to numerical instability or divergence. Raising the order of the differential transform (n) can effectively eliminate this issue, which enhances the precision of the series approximation. When the solution curve exhibits rapid variation, higher-order terms become essential for accuracy. Similarly, using a smaller time step improves the approximation and stability of the solution. In the present study, the domain length $L = 1$ is discretized over 30 equal spatial intervals, with a time interval $\Delta t = 0.001$, and the order of differential transform is taken as $K = 30$. These parameter choices ensure accurate and stable results for both linear and nonlinear cases.

The numerical accuracy of the proposed hybrid method is strongly influenced by the spatial discretization size, time-step interval, and truncation order of the differential transform series. A smaller time-step and higher transform order generally improve solution accuracy and convergence stability. However, excessively fine spatial discretization may increase computational cost. Therefore, appropriate selection of numerical parameters is necessary to achieve a balance between computational efficiency and solution accuracy.

To further verify the reliability of the proposed hybrid DTM–FDM approach, the numerical results were compared with previously published results available in the literature. The comparison demonstrated good agreement, thereby confirming the accuracy and stability of the proposed method for solving both linear and nonlinear transient heat conduction problems.

7. Conclusion

A hybrid Differential Transformation–Finite Difference Method (DTM–FDM) was formulated to analyze linear and nonlinear forms of one-dimensional transient heat conduction problems involving both constant and temperature-dependent thermal conductivity. The simulation results for both linear and nonlinear cases were illustrated. In the linear case ($\beta = 0$), the results demonstrated excellent agreement with expected physical behavior, showing a smooth temperature gradient from the heated end to the cooled end, with convergence achieved using a limited number of series terms.

In the nonlinear cases ($\beta \neq 0$), simulations revealed that increasing β significantly accelerates heat propagation due to the enhanced thermal conductivity at higher temperatures. This was clearly reflected in the time–temperature variation at $x = 0.5$, where higher β values led to faster temperature rise and higher steady-state values. For all considered cases, the Differential Transform Method offers a systematic framework for solving the equation, thereby confirming that the hybrid DTM–FDM technique is an effective and reliable computational framework for handling both linear and nonlinear heat conduction phenomena.

The results confirm that the hybrid DTM–FDM framework is capable of effectively handling nonlinear heat transfer behavior while maintaining numerical stability and convergence accuracy. Therefore, the proposed methodology can serve as a reliable computational approach for solving transient heat conduction problems arising in engineering and applied sciences.

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