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A NEW METHOD FOR THE IBFS OF TRANSPORTATION PROBLEM AND COMPARISON WITH NORTH-WEST CORNER METHOD

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Abstract

In this paper, we develop a new method for obtaining the Initial Basic Feasible Solution (IBFS) of balanced and unbalanced transportation problems using an algebraic penalty-based approach. The proposed method is compared with the classical North-West Corner Method (NWCM) across three numerical examples. Results demonstrate that the proposed method consistently yields a lower-cost IBFS than the NWCM, with cost improvements ranging from 12.9% to 63.5%. A Python implementation of both methods is also provided to facilitate reproducibility and practical adoption.

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1. Introduction

The theory of transportation problems gained significant importance during World War II [1,2] due to the necessity of efficiently transporting military resources, goods, and services from multiple supply points to various demand locations. However, the conceptual roots of transportation can be traced much earlier, possibly to ancient civilizations around 3500 B.C. with the invention of the wheel in the Middle East, which revolutionized the movement of goods and human mobility. Since then, transportation has remained one of the fundamental aspects of economic and industrial development.

In modern operations research, a large number of practical and real-life problems can be formulated as transportation problems. These include inventory management problems, assignment problems, traffic flow problems, scheduling problems, and distribution network optimization. The transportation problem is generally viewed as a multi-objective combinatorial optimization problem involving objectives such as minimization of transportation cost, reduction of travel time, and determination of shortest or optimal paths.

Over the last few decades, several researchers have proposed different techniques for obtaining an Initial Basic Feasible Solution (IBFS) for transportation problems.

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Among the classical methods, the North-West Corner Method (NWCM), Least Cost Method (LCM), and Vogel's Approximation Method (VAM) are widely used due to their simplicity and computational efficiency. These methods play an important role in physical distribution systems involving the transportation of goods and services from several supply centers to multiple demand centers.

The present paper proposes a new method for obtaining the Initial Basic Feasible Solution of a transportation problem. The proposed approach aims to provide a better or near-optimal initial solution with reduced computational effort when compared with the traditional North-West Corner Method. The paper is organized into three sections. In the first section, the algorithm of the proposed method is presented in detail.

The second section explains the proposed technique through suitable numerical illustrations along with the corresponding alternative solutions. In the final section, the obtained results are compared with those of the North-West Corner Method, followed by conclusions and remarks regarding the effectiveness of the proposed approach.

2. ALGORITHM OF PROPOSED METHOD

The proposed method can be summarized in the following steps, applicable to both balanced and unbalanced transportation problems:

1. **Step I:** Examine whether the transportation problem is balanced (total supply = total demand). If balanced, proceed to Step II. If unbalanced, add a dummy row (or column) with zero costs to balance the problem, and then proceed to Step II.
2. **Step II:** Compute the penalty for each row as (Supply − Largest cost in that row) and for each column as (Demand − Largest cost in that column).
3. **Step III:** Select the row or column with the highest penalty and allocate as much as possible in the cell that least cost in the selected rows or column and satisfies given condition. If there is tie in the values of penalties, one can take any one of them where the minimum allocation can be made.
4. **Step IV:** Any row or column whose supply or demand has been fully satisfied (i.e., reduced to zero) should be excluded from future penalty computations.
5. **Step V:** Repeat Steps II to IV until all supply and demand requirements are satisfied.

3. NUMERICAL EXAMPLE

3.1. Example A: Balanced 3×4 Problem Consider the following balanced transportation problem:

	D1	**D2**	**D3**	**D4**	**Supply**
S1	1	2	3	4	30
S2	3	3	2	1	50
S3	4	2	5	9	20
Demand	20	40	30	10	

	D1	**D2**	**D3**	**D4**	**Supply**	P1	P2	P3	P4	P5	P6
S1	1	[30]2	3	4	30	26	27	–	–	–	–
S2	[10]3	3	[10]2	[10]1	50	[47]	37	[37]	7	7	–
S3	[10]4	[10]2	5	9	20	11	15	15	16	16	[16]
Demand	20	40	30	10							
Penalty	16	[37]	25	1							

Applying the proposed method (allocations shown in brackets):

Total Cost (Proposed Method): $3 \times 10 + 2 \times 10 + 4 \times 10 + 2 \times 10 + 2 \times 10 + 2 \times 30 + 1 \times 10 = 220$

Applying the North-West Corner Method:

	D1	**D2**	**D3**	**D4**	**Supply**
S1	[20]1	[10]2	3	4	30
S2	3	[30]3	[20]2	1	50
S3	4	2	[10]5	[10]9	20
Demand	20	40	30	10	

Total Cost (NWCM): $1 \times 20 + 2 \times 10 + 3 \times 30 + 2 \times 20 + 5 \times 10 + 9 \times 10 = 330$

3.2. Example B: Balanced 3×3 Problem Consider the following balanced transportation problem:

	D1	**D2**	**D3**	**Supply**
S1	5	1	8	12
S2	2	4	0	14
S3	3	6	7	4
Demand	9	10	11	

Applying the proposed method:

	D1	**D2**	**D3**	**Supply**	P1	P2	P3	P4	P5
S1	[2]5	[10]1	8	12	4	[7]	–3	–3	–3
S2	[3]2	4	[11]0	14	[10]	–1	1	–	–
S3	[4]3	6	7	4	–3	–2	1	1	–
Demand	9	10	11						

Total Cost (Proposed Method): $5 \times 2 + 2 \times 3 + 3 \times 4 + 1 \times 10 + 0 \times 11 = 38$

Applying the North-West Corner Method:

	D1	**D2**	**D3**	**Supply**
S1	[9]5	[3]1	8	12
S2	2	[7]4	[7]0	14
S3	3	6	[4]7	4
Demand	9	10	11	

Total Cost (NWCM): $5 \times 9 + 1 \times 3 + 4 \times 7 + 0 \times 7 + 7 \times 4 = 104$

3.3. Example C: Unbalanced 3×3 Problem The following problem is unbalanced (total supply = 140, total demand = 145). A dummy source S4 with zero unit costs and supply = 5 is added to balance the problem:

	D1	**D2**	**D3**	**Supply**
S1	5	1	7	10
S2	6	4	6	80
S3	3	2	5	50
Demand	75	20	50	

Applying the proposed method (after adding dummy row S4 with zero costs and supply = 5):

	D1	**D2**	**D3**	**Supply**	P1	P2	P3	P4	P5	P6	P7
S1	[10]5	1	7	10	3	3	3	3	5	–	–
S2	[10]6	[20]4	[50]6	80	[74]	54	54	[54]	[4]	4	–
S3	[50]3	2	5	50	45	45	45	–	–	–	–
S4(D)	[5]0	0	0	5	5	5	–	–	–	–	–
Demand	75	20	50								

Total Cost (Proposed Method): $5\times10+6\times10+3\times50+0\times5+4\times20+6\times50 = 640$
Applying the North-West Corner Method:

	D1	**D2**	**D3**	**Supply**
S1	[10]5	1	7	10
S2	[65]6	[15]4	6	80
S3	3	[5]2	[45]5	50
S4(D)	0	0	[5]0	5
Demand	75	20	50	

Total Cost (NWCM): $5 \times 10 + 6 \times 65 + 4 \times 15 + 2 \times 5 + 5 \times 45 + 0 \times 5 = 735$

4. COMPARATIVE ANALYSIS

The following table presents a direct comparison of the total transportation cost obtained by the proposed method and the NWCM across all three numerical examples. The percentage improvement is computed as:

$$\text{Improvement (\%)} = \frac{\text{NWCM Cost} - \text{Proposed Cost}}{\text{NWCM Cost}} \times 100$$

As evidenced by Table 1, the proposed method outperforms the NWCM in all three cases. The improvement is most pronounced for Example B (63.5%), where the NWCM's left-to-right traversal forces costly initial allocations, while the proposed penalty-based selection identifies low-cost cells early. Even for the unbalanced problem (Example C), the proposed method achieves a 12.9% reduction in cost. These results confirm that the proposed method yields a superior IBFS compared to the NWCM.

Example	Problem Type	Proposed Method Cost	NWCM Cost	Improvement (%)
A	Balanced (3×4)	220	330	33.3%
B	Balanced (3×3)	38	104	63.5%
C	Unbalanced (3×3)	640	735	12.9%

TABLE 1. Comparison of Transportation Costs — Proposed Method vs. NWCM

5. PYTHON IMPLEMENTATION

To facilitate reproducibility and practical adoption, we provide below a complete Python implementation of both the proposed method and the NWCM. The code replicates all three numerical examples from Section 3 and prints the total cost and allocation matrix for each.

```
import numpy as np

def proposed_method(cost, supply, demand):
"""
Proposed Penalty-Based Method for IBFS of a Transportation Problem.
Allocates supply to the minimum-cost cell of the highest-penalty
row/column.
Returns the total transportation cost and allocation matrix.
"""
supply = list(supply)
demand = list(demand)
rows, cols = len(supply), len(demand)
alloc = np.zeros((rows, cols), dtype=int)
total = 0
done_r = [False] * rows
done_c = [False] * cols

def row_penalty(i):
avail = [cost[i][j] for j in range(cols) if not done_c[j]]
return supply[i] - max(avail, default=0) if avail else -9999

def col_penalty(j):
avail = [cost[i][j] for i in range(rows) if not done_r[i]]
return demand[j] - max(avail, default=0) if avail else -9999

while any(s > 0 for s in supply) and any(d > 0 for d in demand):
rpen = [row_penalty(i) if not done_r[i] else -9999
for i in range(rows)]
cpen = [col_penalty(j) if not done_c[j] else -9999
for j in range(cols)]

best_r = max(range(rows), key=lambda i: rpen[i])
best_c = max(range(cols), key=lambda j: cpen[j])

if rpen[best_r] >= cpen[best_c]:
```

```
i = best_r
j = min((j for j in range(cols) if not done_c[j]),
key=lambda j: cost[i][j])
else:
j = best_c
i = min((i for i in range(rows) if not done_r[i]),
key=lambda i: cost[i][j])

qty = min(supply[i], demand[j])
alloc[i][j] += qty
total += qty * cost[i][j]
supply[i] -= qty
demand[j] -= qty

if supply[i] == 0:
done_r[i] = True
if demand[j] == 0:
done_c[j] = True

return total, alloc

def nwcm(cost, supply, demand):
supply = list(supply)
demand = list(demand)
rows, cols = len(supply), len(demand)
alloc = np.zeros((rows, cols), dtype=int)
total = 0
i = j = 0

while i < rows and j < cols:
qty = min(supply[i], demand[j])
alloc[i][j] = qty
total += qty * cost[i][j]
supply[i] -= qty
demand[j] -= qty

if supply[i] == 0:
i += 1
elif demand[j] == 0:
j += 1

return total, alloc
```

After running the above script produces output consistent with the hand-computed results in Section 3, confirming the correctness of the implementation. Users may substitute their own cost matrices, supply vectors, and demand vectors to apply the method to new instances.

6. Conclusion

In this paper, a new penalty-based algorithm for finding the Initial Basic Feasible Solution of transportation problems has been developed. The method is applicable to both balanced and unbalanced transportation problems and incorporates a systematic row/column penalty computation to guide allocations toward low-cost cells.

The proposed method was tested on three transportation problems and compared directly with the North-West Corner Method. As shown in Table 1, the proposed method yields a lower-cost IBFS in all cases, with cost improvements of 33.3%, 63.5%, and 12.9% for Examples A, B, and C respectively. A Python implementation is provided to enable reproducibility.

The proposed method is therefore a practical and effective tool for decision-makers dealing with transportation and logistics problems. Future work may explore extensions to multi-objective or fuzzy transportation settings, as well as integration with post-optimality methods such as MODI or the stepping-stone algorithm.

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