GANITA, Vol. 76(1), 2026, 87–98

EOQ MODEL FOR NON-DECREASING DEMAND OF DETERIORATING ITEMS WITH COMPLETELY BACKORDERING UNDER TWO LEVEL TRADE CREDITS WITH SALVAGE VALUE

PRIYANKA MISHRA ✉

Abstract

Now-a-days supplier facing competitions in business offer trade credit to retailer and retailer himself offer credit to customers. Trade credits are permissible delay in payment by retailer to supplier and by customers to retailer. Salvage value for deteriorating item is considered. Salvage value is taken as cost of items after all depreciations allowed and completing working life. Demand of items is considered non-decreasing with time and complete backordering is allowed. In this paper two cases have been discussed. Optimal solution using calculus and approximation of second order i.e. taking the terms upto second degree in exponential and binomial expansions is obtained. Taking numerical solutions concave graphs are shown to validate the proposed model for maximum profit and the impact of parameters on decision variables are finally investigated.

2010 *Mathematics subject classification:* 90B05.

Keywords and phrases: Permissible delay, Trade credit, deterioration and salvage value.

1. Introduction

Firstly, Harris [1] introduced the classical economic order quantity (EOQ) model, and its extension is done by researchers in several ways. One important issue in this model is delay in payment as an incentive or credit for retailers by suppliers and customers by retailers. Molamohamadi et.al [2] classified different types of delay in payment as pay as sold, pay as sold during given period, pay after given period and pay after at the time of next consignment order. The incentive as delay in payment for retailer is beneficial as he does not need to invest his capital in inventory and can earn interest for items he sells. This pattern of business for supplier is agreement as sales promotional tool for attracting new retailer and customer for selling new and unproven items. Goel [3] EOQ model is basic literature for credit period offered by supplier to retailer. Aggarwal and Jaggi [4], Jamal.et.al [5], Teng [6], Chung and Huang [7] extended the model of Goel [3] considering revenue and interest during credit periods including shortages. Huang [8] considered two-level trade credits. In two- level trade credits not only supplier offer trade credit to retailer but retailer also offers credit period to customer. Chen and

Teng [9] and chen.et.al [10] extended Goel [3] considering interest paid and gain by retailer. Zohareh Molamohamadi.et.al [11] considered EOQ model with completely backordering and non-decreasing demand under two-level trade credits. C.K Jaggi and S.P Aggarwal [12] considered EOQ model for deteriorating items with salvage value. S. Tiwari, et al [13], M. Kumari, et al [14] and P.K. Gupta et al [15] have recently considered EOQ models for deteriorating items with trade credits. P. Mondal et al. [16] considered an EOQ model for deteriorating item with continuous linear time dependent demand with trade credit and replenishment time being demand dependent. D.K. Jana et al. [17] studied a memory dependent partial backlogging inventory model for non-instantaneous deteriorating with stock dependent demand. P. Mahata et al.[18] discussed an economic order quantity model with two-level partial trade credit for time varying deteriorating items. This paper is extension of Z. Molamohamadi.et.al [11] using salvage value for deteriorating items.

1.1 Research Gap and Contribution

Reviewing a brief study of various literature, several researchers have studied EOQ models under several conditions. A comprehensive review of literature related to the proposed model has been shown in table as :

Authors	**Deteriorating items**	**Backordering**	**Allowable shortages**	**Trade Credits**	**Salvage**
S.K. Goel et al. [3]				✓	
S.P. Agarwal et al. [4]	✓			✓	
H.M.M. Jamal et al. [5]	✓		✓	✓	
J.T. Tong et al. [6]				✓	
K.J. Chung et al. [7]				✓	
S.C. Chen et al. [9]	✓			✓	
Z. Molamohamadi et al. [11]		✓		✓	
C.K. Jaggi et al. [12]					✓
S. Tiwari et al. [13]	✓			✓	
M. Kumari et al. [14]	✓			✓	
P.K. Gupta et al. [15]	✓		✓		
P. Mondal et al. [16]	✓			✓	
D.K. Jana et al. [17]	✓	✓			
P. Mahata et al. [18]				✓	
Our work	✓	✓		✓	✓

2. Notations

c_1 : Ordering cost per order.
c_2 : Purchasing cost per unit.
c_3 : Inventory carrying cost per unit per unit of time.
c_4 : Cost of each deteriorated unit.
s : Selling cost per unit ($s > c_2$)

s_1: Shortage cost per unit per unit time.
h : Stock holding cost per unit items and per unit time leaving interest charges.
I_1 : Interest earned by retailer per item per unit of time.
I_2 : Interest charged by supplier per item per unit of time.
M_1: Trade credit period in days offered by supplier to retailer
M_2: Trade credit period in days offered by retailer to customers.
P : Net profit of retailer per unit time.
T : Time of inventory cycle.
T_1 : Cycle time of inventory having present in stock.
Q : Ordered quantity of retailer.
Q_1 : Inventory depleted in time T_1.
θ : Deterioration rate of items ($0 \leq \theta < 1$)
kc_2 : Salvage value associated to deteriorated units of inventory during one complete cycle, 0≤k<1.

3. Assumptions

1. Demand is non- decreasing function and depends linearly on time i.e.

$$D(t) = a + bt, \quad \text{where a>0, b>0.}$$

2. Shortages are permitted and completely backordered.
3. Replenishment arises immediately at endless pace.
4. Suppliers offer much delay period M_1 for payment to retailer and retailer provides shorter period M_2 to customer for payment. The retailer at the end of credit period M_1 pays off to supplier and pays interest on remaining items in his stock with rate I_2 in interval $[M_1, T_1]$ where ($T_1 > M_1$). If credit period is greater than positive stock replenishment cycle then retailer is not charged any interest in settling the amount.
5. The retailer collect the revenue and earns interest with rate I_1 from M_2 to M_1.

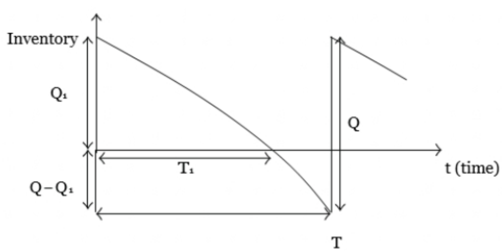

FIGURE 1. Inventory level of retailer from 0 to T.

4. Mathematical Formulation and calculation

Following notations and assumptions of sec-2 and sec-3, the retailers inventory system depicted in Fig (1) is explained as follows. Retailer orders and receive quantity Q units of items at t=0 and supply previously backlogged orders to the customers immediately.The remaining inventory Q_1 depletes gradually due to deterioration and demand of customers and become 0 at $t = T_1$. From T_1 to T the retailer has no inventory in stock and arriving orders from customers are fully backlogged for next cycle. The items in this inventory model may be tremendously changing fashion wares and quality-based edible goods. The inventory level I at any instant 't' is governed by differential equations

$$\frac{d}{dt}I(t) + \theta I(t) = -(a + bt), \qquad 0 \le t \le T_1, \tag{4.1}$$

$$\frac{d}{dt}I(t) = -(a + bt), \qquad T_1 \le t \le T, \tag{4.2}$$

$$\text{With boundary conditions,} \quad I(T_1) = 0. \tag{4.3}$$

Solution of equations (4.1) and (4.2) using (4.3) are

$$I(t) = \frac{1}{\theta}\left\{(a\theta - b)\frac{\left(e^{\theta(T_1 - t)} - 1\right)}{\theta} + b\left(T_1 e^{\theta(T_1 - t)} - t\right)\right\}. \tag{4.4}$$

$$I(t) = (T_1 - t)\left\{a + \frac{b}{2}(T_1 + t)\right\}. \tag{4.5}$$

Inventory level at beginning of replenishment cycle is (putting t=0 in equation (4.4))

$$Q_1 = I(0) = \frac{bT_1 e^{\theta T_1}}{\theta} + \frac{(a\theta - b)}{\theta^2}\left(e^{\theta T_1} - 1\right). \tag{4.6}$$

$$-(Q - Q_1) = I(T) = (T_1 - T)\left\{a + \frac{b}{2}(T + T_1)\right\}. \tag{4.7}$$

Since there are no units in inventory at t = T_1. Then, retailer is allowed to back ordering during time T_1 to T.

for, $T_1 <$ T, I(T) = Q_1- Q = -($Q - Q_1$) < 0.

This negative sign indicates inventory is not in stock and demand is backordered. Now, retailer's order size per cycle time is given by

$$\begin{aligned} Q &= I(0) + |\,I(T)| \\ &= \frac{bT_1 e^{\theta T_1}}{\theta} + \frac{(a\theta - b)}{\theta^2}(e^{\theta T_1} - 1) + |\,(T_1 - T)\{a + \frac{b(T_1 + T)}{2}\}| \\ &= \frac{bT_1 e^{\theta T_1}}{\theta} + \frac{(a\theta - b)}{\theta^2}(e^{\theta T_1} - 1) + (T - T_1)\{a + \frac{b(T_1 + T)}{2}\}. \end{aligned} \tag{4.8}$$

Ordering cost per order say **OC** = c_1. (4.9)

Total demand during [0, T_1] is :

$$\int_0^{T_1} D(t)\,dt = \int_0^{T_1} (a+bt)dt = T_1\,(a+\frac{b}{2}T_1). \quad (4.10)$$

Deteriorated Number of units = Maximum inventory- Demand during [0, T_1] is :

$$Q_1 - T_1(a+\frac{b}{2}T_1) = \frac{bT_1e^{\theta T_1}}{\theta} + \frac{(a\theta-b)}{\theta^2}(e^{\theta T_1}-1) - T_1(a+\frac{b}{2}T_1). \quad (4.11)$$

Deterioration cost (DC) during [0, T_1] is given by :

$$c_4\,\{\frac{b\ T_1e^{\theta T_1}}{\theta} + \frac{(a\theta-b)}{\theta^2}(e^{\theta T_1}-1)\ -T_1(a+\frac{b}{2}T_1)\ \} \quad (4.12)$$

Inventory holding cost during [0, T_1] leaving interest charges is :

$$\text{HC} = \text{h}\int_0^{T_1} I(t)dt$$

$$= \text{h}\int_0^{T_1} \frac{1}{\theta}\{(a\theta-b)\frac{\left(e^{\theta(T_1\ -t)}-1\right)}{\theta} \ +\ b\ \left(T_1e^{\theta(T_1-t)} - t\right)\}\,dt$$

$$= \frac{h}{\theta}\{(a-\frac{b}{\theta})\ \frac{(\ e^{\theta T_1}\ -1-\ \theta T_1)}{\theta}\ +\ bT_1\ (\ \frac{(e^{\theta T_1}\ -1)}{\theta}\ -\ \frac{T_1}{2})\ \}$$

$$= \frac{h}{\theta}\{(a-\frac{b}{\theta})\ \frac{(\theta^2{T_1}^2)}{2\theta}\ +\ \frac{bT_1}{\theta}\ (\ \theta T_1\ +\ \frac{\theta^2{T_1}^2}{2})\ \text{-}\ \frac{(b{T_1}^2)}{2}\}$$

$$= \frac{h{T_1}^2}{2}(\ \text{a}+bT_1). \quad (4.13)$$

Shortage cost during [T_1, T] is :

$$\text{-}s_1\ \int_{T_1}^{T} I(t)\ dt$$

$$= \text{-}s_1\int_{T_1}^{T} \{(T_1-t)\ (a+\frac{b}{2}\ (T+\ t_1)\}dt$$

$$= s_1\int_{T_1}^{T} (t-T_1)\ (a+\frac{b}{2}(T+\ t_1))dt$$

$$= s_1\frac{(T-T_1)^2}{6}\{\ 3\text{a}+\text{b}(\ \text{T}+2T_1)\ \}. \quad (4.14)$$

Salvage value in [0, T_1] is :

kC_2 x Number of deteriorated unit during [0, T_2]

$$= \text{k}C_2\{\frac{b\ T1e^{\theta T_1}}{\theta} + \frac{(a\theta-b)}{\theta^2}(e^{\theta T_1}-1)\ \text{-}T_1(a+\frac{b}{2}T_1)\ \}$$

$$= \text{k}C_2\ \{\ \frac{bT_1}{\theta}\ (1+\theta T_1 + \frac{\theta^2{T_1}^2}{2}\ ) + \frac{(a\theta-b)}{\theta^2}\ (\ \theta T_1 + \frac{\theta^2{T_1}^2}{2}\ )\ \text{- a}\ T_1\ \text{-}\ \frac{b}{2}\ T_1^2\}$$

$$= \text{k}C_2\ (\frac{a}{2}\ \theta\ T_1^2 + \frac{b}{2}\ \theta\ T_1^3)$$

$$= \text{k}C_2\theta\frac{T_1^2}{2}(\ a+bT_1). \quad (4.15)$$

Sales revenue in [0, T] is :

sQ = s{ $\frac{b\,T1e^{\theta T_1}}{\theta}$ + $\frac{(a\theta-b)}{\theta^2}$($e^{\theta T_1}$-1) + (T-T_t1) (a + $\frac{b(T1+T)}{2}$)}

= s { $\frac{bT_1}{\theta}$ (1+ θT_1 + $\frac{\theta^2 T_1{}^2}{2}$) + $\frac{(a\theta-b)}{\theta^2}$ (θT_1 + $\frac{\theta^2 T_1{}^2}{2}$) + a($T - T_1$) + $\frac{b}{2}$($T^2 - T\_1^2$)}

= s (a T + $\frac{b}{2}T^2$ + $\frac{a}{2}\,\theta\,T_1^2$ + $\frac{b}{2}\,\theta\,T_1^3$). (4.16)

Payable interest

For calculating interest paid, we consider two cases.

Case 1 ($T_1 \le M_1$) :delay period is greater than or equal to positive stock time.

In this case retailer does not have any stock in hand. Now interest paid in this case is :

zero.

Case 2 ($M_1 \le T_1$): delay period is less than or equal to positive stock time.

In this case retailer is charged interest for on-hand inventory between M_1 and T_1 with rate I_2. Interest payable is

$= c_2 I_2 \int_{M_1}^{T_1} \frac{1}{\theta}\{(a\theta - b)\frac{\left(e^{\theta(T_1\ -t)}-1\right)}{\theta}\ +\ b\ \left(T_1 e^{\theta(T_1-t)} - t\right)\}\, dt$

$= c_2 I_2$ { $\frac{(a\theta-b)\ +\ b\theta T_1}{\theta^3}$($e^{\theta(T_1-M_1)} - 1) - \frac{(T_1-M_1)}{\theta}$($\frac{a\theta-b}{\theta}$ + $\frac{b}{2}(T_1 + M_1)$}

$= c_2 I_2${ $\frac{(a\theta-b)\ +\ b\theta T_1}{\theta^3}$ ($\theta(T_1 - M_1)$ + $\frac{\theta^2}{2}(T_1 - M_1)^2) -$ $\frac{(a\theta-b)}{\theta^2}(T_1 - M_1)$ − $\frac{b(T_1^2-M_1^2)}{2\theta}$}

$= c_2 I_2$ { $\frac{b}{\theta}T_1^2$ − $\frac{b}{\theta}M_1^2$ + $\frac{(a+bT_1)}{2}$ ($T_1^2 -$ $2T_1M_1$ + M_1^2) }. (4.17)

Interest Earned

Retailer earns interest for sold items between M_2 to M_1.

Now we consider two cases :

Case (1) ($M_1 - M_2$) $\le$ T_1.

In this case retailer gets the payment for $Q - Q_1$ items from customer at time M_2 and after that payment is settled with rate a+bt for forward time Revenue function R(t) in interval [$M_2, M_2 + T_1$] is given by

$$\frac{d}{dt}I(t) = (a + bt)\ M_2(\le t \le)M_2{+}T_1, \qquad (4.18)$$

with R(M_2) =0. (4.19)

Solution of revenue equations (4.18) with boundary condition (4.19) is :

$$R(t) = a(t - M_2) + \frac{b(t^2-M_2^2)}{2}. \qquad (4.20)$$

Interest earned per cycle when ($M_1 - M_2$)($\le$)T_1 is :

$sI_1\{\int_{M_2}^{M_1} R(t)dt\ \ +\ (Q - Q_1)(M_1 - M_2)\}$

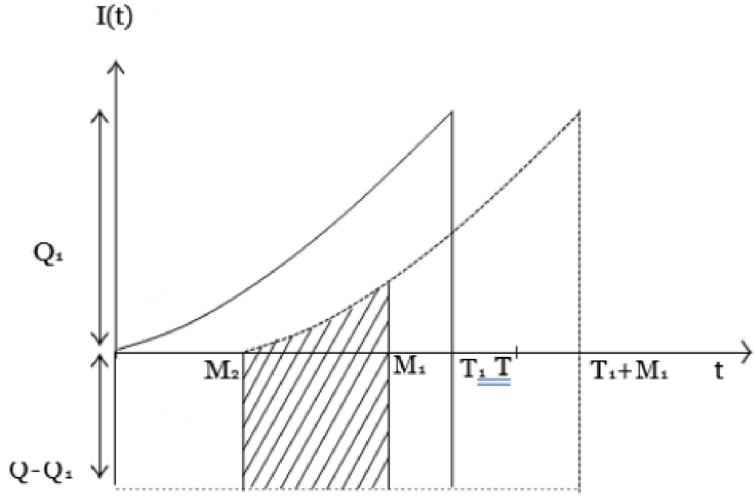

FIGURE 2. Accumulation of interest earned when $M_1 - M_2 \le T_1$

$$= sI_1 \left\{ \int_{M_2}^{M_1} \{a(t - M_2) + \frac{b(t^2-M_2^2)}{2}\}dt + (Q-Q_1)(M_1-M_2)\right\}$$

$$= sI_1\{ \frac{a}{2}(M_1-M_2)^2 + \frac{b(M_1^3-M_2^3)}{6} - \frac{b}{2}M_2^2(M_1-M_2) + (T - T_1)(M_1-M_2)(a+\frac{b(T+T_1)}{2}\}. \quad (4.21)$$

Case (2) $(M_1 - M_2) \ge T_1$.

Retailer's earned interest in this case is :

$$sI_1\{\int_{M_2}^{T_1+M_2} R(t)dt + (Q-Q_1)(M_1-M_2) + Q_1(M_1-T_1-M_2)\}$$

$$= sI_1 \left\{ \int_{M_2}^{T_1+M_2} (a(t - M_2) + \frac{b(t^2-M_2^2)}{2})dt + (Q-Q_1)(M_1-M_2) + Q_1(M_1-T_1-M_2)\right\}$$

$$=sI_1\{ \frac{a}{2}(T_1)^2 + \frac{b((T_1+M_2)^3-M_2^3)}{6} - \frac{b}{2}M_2^2(T_1) + (T-T_1)(M_1-M_2)\left(a+\frac{b(T+T_1)}{2}\right)$$
$$+(M_1-T_1-M_2)(\frac{bT_1e^{\theta T_1}}{\theta} + \frac{(a\theta-b)}{\theta^2}(e^{\theta T_1}-1))\}$$

$$= sI_1\{ \frac{a}{2}(T_1)^2 + \frac{b((T_1+M_2)^3-M_2^3)}{6} - \frac{b}{2}M_2^2(T_1) + (T-T_1)(M_1-M_2)\left(a+\frac{b(T+T_1)}{2}\right)$$
$$+ (M_1-T_1-M_2)(\frac{bT_1}{\theta}(1+\theta T_1 + \frac{\theta^2T_1^2}{2}) + \frac{(a\theta-b)}{\theta^2}(\theta T_1 + \frac{\theta^2T_1^2}{2})\}$$

$$=sI_1\{ \frac{a}{2}(T_1)^2 + a(T-T_1)(M_1-M_2) + aT_1\left(1+\frac{\theta T_1}{2}\right)(M_1-T_1-M_2) + \frac{b((T_1+M_2)^3-M_2^3)}{6}$$

$$-\frac{b}{2}M_2^2(T_1) + \frac{b(T^2-T_1^2)(M_1-M_2)}{2} + (M_1-T_1-M_2)\frac{b}{2}(T_1)^2(1+\theta T_1)\}$$

. (4.22)

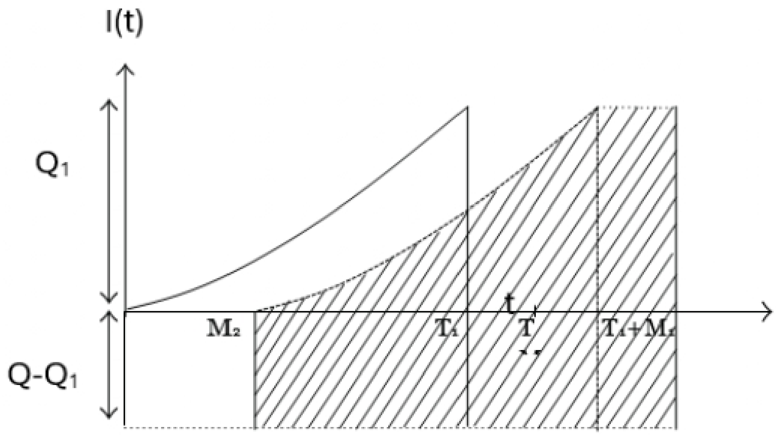

FIGURE 3. Accumulation of interest earned when $(M_1 - M_2) \geq T_1$.

Retailer's net profit per unit time is calculated as:

$\frac{1}{T}$ [Revenue -Ordering cost - Purchasing cost - Holding cost - Shortage cost - Interest payable + Interest earned + Solvage value]

Now,
$$P[T_1 - T] = \begin{cases} P_1, for M_1 \leq T_1, \\ P_2, for T_1 \leq M_1 \leq (T_1 + M_1) \\ P_3, for M_1 \geq (T_1 + M_2). \end{cases}$$

$$P_1 = \frac{(S-C_2)}{T}\{aT + \frac{bT^2}{2} + \frac{aT_1^2\theta}{2} + \frac{bT_1^3\theta}{2}\} - \frac{C_1}{T} - \frac{hT_1^2}{2T}(a + bT_1)$$

$$-\frac{s_1}{T}\{\frac{a}{2}(T^2 - 2TT_1 + T_1^2) + \frac{b}{6}(T^3 - 2T^2T_1 + T_1^2T) + \frac{b}{3}(T_1T^2 - 2TT_1^2 + T_1^3)\}$$

$$-\frac{c_2I_2}{T}\{\frac{b}{\theta}T_1^2 - \frac{b}{\theta}M_1^2 + \frac{(a + bT_1)}{2}(T_1^2 - 2T_1M_1 + M_1^2)\}$$

$$+ \frac{SI_1}{T}\{\frac{a}{2}(M_1 - M_2)^2 + \frac{b}{6}(M_1^3 - M_2^3) - \frac{b}{2}M_2^2(M_1 - M_2) + (T - T_1)(M_1 - M_2)(a + \frac{b}{2}(T + T_1)\}$$

$$+\frac{kC_2\theta T_1^2}{2T}(a+bT_1) \tag{4.23}$$

$$P_2 = \frac{(S-C_2)}{T}\{aT + \frac{bT^2}{2} + \frac{aT_1^2\theta}{2} + \frac{bT_1^3\theta}{2}\} - \frac{C_1}{T} - \frac{hT_1^2}{2T}(a + bT_1)\}$$

$$-\frac{s_1}{T}\{\frac{a}{2}(T^2 - 2TT_1 + T_1^2) + \frac{b}{6}(T^3 - 2T^2T_1 + T_1^2T) + \frac{b}{3}(T_1T^2 - 2TT_1^2 + T_1^3)\}$$

$$+\frac{SI_1}{T}\{\frac{a}{2}(M_1 - M_2)^2 + \frac{b}{6}(M_1^3 - M_2^3) - \frac{b}{2}M_2^2(M_1 - M_2) + (T - T_1)(M_1 - M_2)(a + \frac{b}{2}(T + T_1)\}$$

$+ \frac{kC_2\theta T_1^2}{2T}(a+bT_1)$ (4.24)

$P_3 = \frac{(S-C_2)}{T}\{aT + \frac{bT^2}{2} + \frac{aT_1^2\theta}{2} + \frac{bT_1^3\theta}{2}\} - \frac{C_1}{T} - \frac{hT_1^2}{2T}(a + bT_1)$

$-\frac{s_1}{T}\{\frac{a}{2}(T^2 - 2TT_1 + T_1^2) + \frac{b}{6}(T^3 - 2T^2T_1 + T_1^2T) + \frac{b}{3}(T_1T^2 - 2TT_1^2 + T_1^3)\}$

$+ \frac{SI_1}{T}\{\frac{a}{2}T_1^2 + a(T - T_1)(M_1 - M_2) + aT_1(1 + \theta\frac{T_1}{2})(M_1 - T_1 - M_2)$

$+ \frac{b}{6}((T_1 + M_2)^3 - M_2^3) - \frac{bM_2^2T_1}{2} + \frac{b(T^2-T_1^2)(M_1-M_2)}{2} + (M_1 - T_1 - M_2)\frac{bT_1^2}{2}(1 + \theta T_1)\}$

$+ \frac{kC_2\theta T_1^2}{2T}(a + bT_1).$ (4.25)

Extreme value of P_1 with reference to T and T_1 is obtained by $\frac{\partial P_1}{dT} = 0$ and $\frac{\partial P_1}{dT_1} = 0$

$\frac{\partial P_1}{dT} = -\frac{(S-C_2)}{T^2}(aT + \frac{bT^2}{2} + \frac{aT_1^2\theta}{2} + \frac{bT_1^3\theta}{2}) + \frac{(S-C_2)(Tb+a)}{T} + \frac{C_1}{T^2} + \frac{hT_1^2}{2T^2}(a + bT_1)$

$+\frac{S_1}{T^2}\{\frac{a}{2}(T^2 - 2TT_1 + T_1^2) + \frac{b}{6}(T^3 - 2T^2T_1 + T_1^2T) + \frac{b}{3}(T_1T^2 - 2TT_1^2 + T_1^3)\}$

- $\frac{S_1}{T}\{a\frac{(2T-2T_1)}{2} + b\frac{(3T^2-4TT_1+T_1^2)}{6} + b\frac{(2TT_1-2T_1^2)}{3}\}$

$+\frac{C_2I_2}{T^2}\{\frac{b}{\theta}T_1^2 - \frac{b}{\theta}M_1^2 + \frac{(a+bT_1)}{2}(T_1^2 - 2T_1M_1 + M_1^2)\}$

$-\frac{SI_1}{T^2}\{\frac{a}{2}(M_1 - M_2)^2 + \frac{b}{6}(M_1^3 - M_2^3) - \frac{b}{2}M_2^2(M_1 - M_2) + (T - T_1)(M_1 - M_2)(a + \frac{b}{2}(TT_1)\}$

$+\frac{SI_1}{T}\{(M_1 - M_2)(a + \frac{b}{2}(T + T_1)) + (T - T_1)(M_1 - M_2)\frac{b}{2}\} - \frac{kC_2\theta T_1^2}{2T^2}(a + bT_1)\}=0.$ (4.26)

$\frac{\partial P_1}{dT_1} = \frac{(S-C_2)}{T}\left(a\theta T_1 + \frac{3b\theta T_1^2}{2}\right) - hT_1(a + bT_1) - \frac{hbT_1^2}{2}$

$- \frac{S_1}{T}\{a(T_1\text{-T}) + \frac{b}{6}(2TT_1 - 2T^2) + \frac{b}{3}(T^2 - 4TT_1 + 3T_1^2)\}$

$- \frac{C_2I_2}{T}\{\frac{2b}{\theta}T_1 + \frac{b}{2}(T_1^2 - 2T_1M_1 + M_1^2) + (a+bT_1)(T_1 - M_1)\}$

$+\frac{SI_1}{T}\{-(M_1 - M_2)\left(a + \frac{b}{2}(T + T_1)\right) + (T - T_1)(M_1 - M_2)\frac{b}{2}\}$

$+ \frac{kC_2\theta T_1}{T}(a + bT_1) + \frac{kC_2\theta T_1^2}{2T}a = 0.$ (4.27)

It is difficult to find second order derivative to analyse extreme value for maximum (i.e. rt -s^2 > 0, r < 0) we can show this observation using graph putting approximate value of parameters.

5. Numerical example

Let us consider inventory parameters in approximate units $a = 100, b = 85, M_1 = 3, M_2 = 2, s_1 = 3, I_1 = 3, I_2 = 2, s = 45, c_2 = 20, h = 2, c_1 = 150, (\theta) = 0.5, k = 0.4$. Substituting these parameters in equations (4.26) & (4.27) and solving numerically for T & T_1 We get T= 78 days and T_1= 4 days.

Putting the value of T and T_1 in equation (4.23) we get: P_1= Rs 277990.

Table 1 – Variation of Parameters

For $M_1 = 3$, $M_2 = 2$, $s_1 = 3$, $I_1 = 3$, $I_2 = 2$, $s = 45$, $c_2 = 20$, $h_2 = 2$, $c_1 = 150$, $\theta = 0.5$, $k = 0.4$, we have:

T_1	3	4	6	8	10	12
P_1	81000	82000	83000	84000	83500	83100

Plot of P_1 with respect to $T_1 = 2, \ldots\ldots\ldots.12$ is

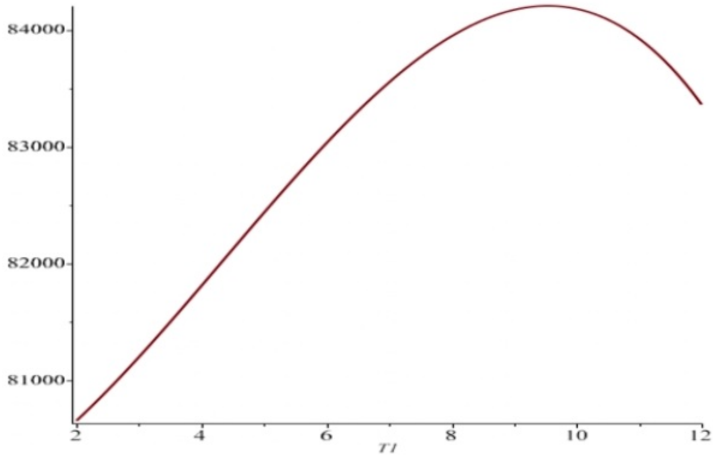

FIGURE 4.

Table 2 – Variation of Parameters

For $M_1 = 3$, $M_2 = 2$, $s_1 = 3$, $I_1 = 3$, $I_2 = 2$, $s = 45$, $c_2 = 20$, $h_2 = 2$, $c_1 = 150$, $\theta = 0.5$, $k = 0.4$, we have:

T	5	20	40	60	80	100	120	140
P_1	50000	100000	200000	250000	230000	220000	170000	80000

Plot of P_1 with respect to T=10,. . . , 150 is

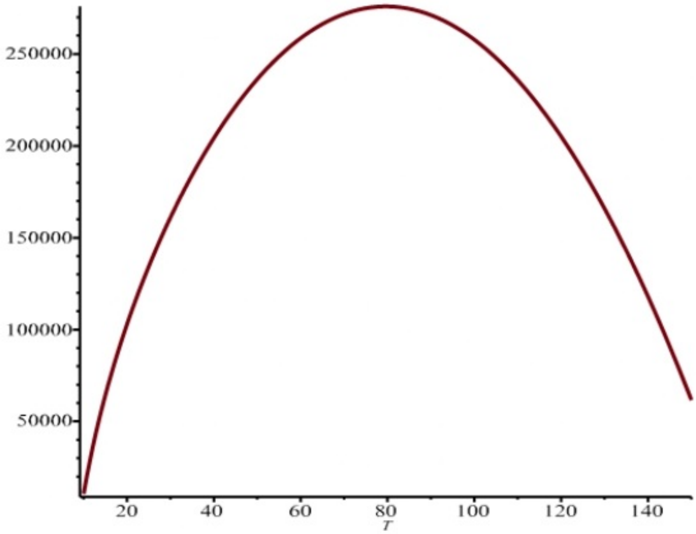

FIGURE 5.

6. Conclusion

In this paper demand rate is considered linearly time dependent and shortages are allowed.Two cases have been considered, Case 1 ($T_1 \leqslant M_1$) and Case 2 ($T_1 > M_1$) .Optimal net profit per unit time is obtained for both Cases. Numerical example is discussed to validate the proposed model. Finally two graphs are drawn to authenticate the considered model by using Maple 2022.

Conflicts of Interest : The authors declare no conflict of interest.

References

[1] F.W. Harris, *How many parts to make at once*, Factory, The Magazine of Management **10** (2) (1913) 135–136.
[2] Z. Molamohamadi, M. Rezaeiahari and N. Ismail, *Consignment inventory: review and critique of literature*, Journal of Basic and Applied Scientific Research **3** (6) (2013) 707–714.
[3] S.K. Goyal, *Economic order quantity under conditions of permissible delay in payments*, Journal of the Operational Research Society **36** (4) (1985) 335–338.
[4] S.P. Aggarwal and C.K. Jaggi, *Ordering policies of deteriorating items under permissible delay in payments*, Journal of the Operational Research Society **46** (5) (1995) 658–662.
[5] A.M.M. Jamal, B.R. Sarker and S. Wang, *An ordering policy for deteriorating items with allowable shortage and permissible delay in payment*, Journal of the Operational Research Society **48** (8) (1997) 826–833.

[6] J.-T. Teng, *On the economic order quantity under conditions of permissible delay in payments*, Journal of the Operational Research Society **53** (8) (2002) 915–918.

[7] K.-J. Chung and Y.-F. Huang, *The optimal cycle time for EPQ inventory model under permissible delay in payments*, International Journal of Production Economics **84** (3) (2003) 307–318.

[8] Y.-F. Huang, *An inventory model under two levels of trade credit and limited storage space derived without derivatives*, Applied Mathematical Modelling **30** (5) (2006) 418–436.

[9] S.-C. Chen and J.-T. Teng, *Retailer's optimal ordering policy for deteriorating items with maximum life time under supplier's trade credit financing*, Applied Mathematical Modelling **38** (15–16) (2014) 4049–4061.

[10] S.-C. Chen, J.-T. Teng and K. Skouri, *Economic production quantity models for deteriorating items with up-stream full trade credit and down-stream partial trade credit*, International Journal of Production Economics **155** (2014) 302–309.

[11] Z. Molamohamadi, R. Arshizadeh, N. Ismail and A. Azizi, *An economic order quantity model with completely backordering and non-decreasing demand under two-level trade credit*, Advances in Decision Sciences **14** (2014).

[12] C.K. Jaggi and S.P. Agrawal, *EOQ for deteriorating items with salvage value*, Bulletin of Pure and Applied Sciences **15E** (1) (1996) 67–71.

[13] S. Tiwari, L. Barron, A.I. Malik and C.K. Jaggi, *Retailer's credit and inventory decisions for imperfect quality and deteriorating items under two-level trade credit*, Computers and Operations Research **138** (2022) 105617.

[14] M. Kumari and P.K. De, *An EOQ model for deteriorating items analyzing retailer's optimal strategy under trade credit and return policy with non-linear demand and resalable returns*, International Journal of Optimization and Control **12** (1) (2022) 47–55.

[15] P.K. Gupta and V.N. Tripathi, *An EOQ model for linearly time dependent deterioration with quadratic time dependent demand and quadratic time dependent shortages*, Tujjin Jishu/Journal of Propulsion Technology **45** (2) (2024).

[16] P. Mondal, A.K. Das and T.K. Roy, *An EOQ model for deteriorating item with continuous linear time dependent demand with trade credit and replenishment time being demand dependent*, International Journal of Mathematics in Operational Research, Vol. **24**, No. 1 (2023).

[17] D.K. Jana and A.K. Das, *A memory dependent partial backlogging inventory model for non-instantaneous deteriorating with stock dependent demand*, International Journal of Applied and Computational Mathematics, Vol.**7**, No. 199, pp. 1–20 (2021).

[18] P. Mahata, G.C. Mahata and S.K. De, *An economic order quantity model with two-level partial trade credit for time varying deteriorating items*, International Journal of System Science: Operations and Logistics, Vol. **7**, No. 1, pp. 1-17 (2020).

Priyanka Mishra, Department of Mathematics and Computer Sciences, Babu Banarasi Das University Lucknow, U.P, India
e-mail: 040396pm@gmail.com