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SOFT IP*-OPEN SETS IN SOFT TOPOLOGICAL SPACES

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Abstract

Topology is an area of mathematics concerned with the properties of space that are preserved under continuous deformation including stretching and bending but not tearing. This paper aims at developing a soft topology via soft IP*-open sets. Further, we study the fundamental properties of soft IP*-open set, soft IP*-closed set, soft IP*-interior, soft IP*-closure, soft IP*-boundary, soft IP*-limit point, soft IP*-derived set and soft IP*-neighborhood.

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1. Introduction

In 1999 Molodtsov [5] presented the fundamental results of the soft set theory and successfully applied it to several directions such as game theory, operation research, theory of probability etc. Soft topology was introduced by Muhammad Shabir et al [6]. In 2011 Shabir Hussain et al. [8] defined and discussed properties of soft interior, soft exterior and soft boundary. Gnanambal llango and Mrudula Ravindran [1] made a study of soft topology via pre-open set. Kannan [3] study on soft g closed set. P. Gnanachandra, M. Lellis Thivagar [10] developing a soft topology via soft pre*-open sets and study the fundamental properties of soft pre*-open, soft pre*-closed sets, soft pre*-interior and soft pre*-closure operators. In this paper we study the fundamental properties of soft IP*-open set, soft IP*-closed set, soft IP*-interior, soft IP*-closure, soft IP*-boundary, soft IP*-limit point, soft IP*-derived set and soft IP*-neighborhood.

Let us recall the following definitions that are used in sequel.

2. Preliminaries

In this segment, we recall some necessary definitions and notations. Also in this research, we express soft operations by '$\sim$', soft IP*-open ($\widetilde{IP}^{*}O$), soft IP*-closed ($\widetilde{IP}^{*}C$). Throughout this entire article, we will refer to U as an initial universal set, *(X, E, τ)* is a soft topological space and *(F,E)* is a soft set over U.

Definition 2.1 [5]. Let U be an initial universal set and E be a set of parameters. Let $P(U)$ denote the power set of U and let $A \subseteq E$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \longrightarrow P(U)$. The collection of soft sets (F, A) over a universe U and the parameter set A is a family of soft sets denoted by $\mathrm{SS}(U)_A$.

Definition 2.2 [6]. Let τ be a collection of soft sets over a universe U with a fixed set A of parameters, then $\tau \subseteq \mathrm{SS}(U)_A$ is called a soft topology on U with a fixed set A if,

(i) Φ_A, U_A belong to τ.
(ii) The union of any number of soft sets in τ belongs to τ.
(iii) The intersection of any two soft sets in τ belongs to τ.

The triplet (U, A, τ) is called a soft topological space over U. The members of τ are called soft open sets in U and complements of them are called soft closed sets in U. Soft operations are denoted by usual set theoretical operations with '$\sim$' symbol above. Soft interior and soft closure are denoted by $\widetilde{s}\mathrm{int}$ and $\widetilde{s}\mathrm{cl}$ respectively.

Definition 2.3 [12]. Let (U, A, τ) be a soft topological space and let (G, A) be a soft set. Then (i) The soft closure of (G, A) is the soft set

$$\widetilde{s}\mathrm{cl}(G, A) = \widetilde{\bigcap}\{(S, A) : (S, A) \text{ is soft closed and } (G, A) \mathrel{\widetilde{\subseteq}} (S, A)\}$$

(ii) The soft interior of (G, A) is the soft set

$$\widetilde{s}\mathrm{int}(G, A) = \widetilde{\bigcup}\{(S, A) : (S, A) \text{ is soft open and } (S, A) \mathrel{\widetilde{\subseteq}} (G, A)\}$$

$\widetilde{s}\mathrm{cl}(G, A)$ is the smallest soft closed set containing (G, A) and $\widetilde{s}\mathrm{int}(G, A)$ is the largest soft open set contained in (G, A).

Definition 2.4 [10]. A subset of a topological space (X, τ) is called pre*-open if $A \subseteq \mathrm{int}^*(\mathrm{cl}(A))$.

Definition 2.5 [3]. Let A be a subset of X. The generalized closure of A is defined as the intersection of all g-closed sets containing A and is denoted by $\mathrm{cl}^*(A)$. The generalized interior of A is defined as the union of all g-open sets contained in A and is denoted by $\mathrm{int}^*(A)$.

Definition 2.6 [10]. In a soft topological space (U, A, τ), a soft set

(i) (G, A) is said to be soft pre-open set if $(G, A) \mathrel{\widetilde{\subseteq}} \widetilde{s}\mathrm{int}(\widetilde{s}\mathrm{cl}(G, A))$.
(ii) (F, A) is said to be soft pre-closed set if $(F, A) \mathrel{\widetilde{\supseteq}} \widetilde{s}\mathrm{cl}(\widetilde{s}\mathrm{int}(F, A))$.

A soft pre-closed set is nothing but the complement of a soft pre-open set.

(i) (G, A) is said to be soft pre*-open set if $(G, A) \mathrel{\widetilde{\subseteq}} \widetilde{s}\mathrm{int}^*(\widetilde{s}\mathrm{cl}(G, A))$
(ii) (F, A) is said to be soft pre*-closed set if $(F, A) \mathrel{\widetilde{\supseteq}} \widetilde{s}\mathrm{cl}^*(\widetilde{s}\mathrm{int}(F, A))$

A soft pre*-closed set is the complement of a soft pre*-open set.

Theorem 2.1 [10]. *Every soft open set is a soft pre-open set.*

Theorem 2.2 [10]. *Every soft open set is a soft pre*-open set.*

Theorem 2.3 [10]. *Arbitrary union of soft pre*-open sets is a soft pre*-open set.*

Theorem 2.4 [10]. *Arbitrary intersection of soft pre*-closed sets is a soft pre*-closed set.*

3. Soft IP*-open sets

In this section, we define soft IP*-open sets in soft topological spaces and study some of their basic properties.

Definition 3.1. In a soft topological space (X, E, τ), a soft set

(i) (M, E) is said to be soft IP*-open set
if $(M, E) \tilde{\subseteq} \{\tilde{s}\mathrm{int}(\tilde{s}\mathrm{cl}(M, E))\} \tilde{\bigcap} \{\tilde{s}\mathrm{int}^*(\tilde{s}\mathrm{cl}(M, E))\}$.

In other words soft IP*-open sets is a soft subset of soft intersection of soft pre-open set and soft pre*-open set.

(ii) (M, E) is said to be soft IP*-closed set
if $(M, E) \tilde{\supseteq} \{\tilde{s}\mathrm{cl}(\tilde{s}\mathrm{int}(M, E))\} \tilde{\bigcup} \{\tilde{s}\mathrm{cl}^*(\tilde{s}\mathrm{int}(M, E))\}$.

Example 3.1. Let $X = \{c_1, c_2\}$, $E = \{t_1, t_2\}$ Define

$$
\begin{array}{ll}
F_1(t_1) = \{c_1, c_2\}, & F_1(t_2) = \{c_2\}, \\
F_2(t_1) = \{c_1, c_2\}, & F_2(t_2) = \{c_1, c_2\}, \\
F_3(t_1) = \{c_1, c_2\}, & F_3(t_2) = \phi, \\
F_4(t_1) = \{c_1, c_2\}, & F_4(t_2) = \{c_1\}, \\
F_5(t_1) = \{c_2\}, & F_5(t_2) = \{c_2\}, \\
F_6(t_1) = \{c_2\}, & F_6(t_2) = \{c_1, c_2\}, \\
F_7(t_1) = \{c_2\}, & F_7(t_2) = \phi, \\
F_8(t_1) = \{c_2\}, & F_8(t_2) = \{c_1\}, \\
F_9(t_1) = \{c_1\}, & F_9(t_2) = \{c_2\}, \\
F_{10}(t_1) = \{c_1\}, & F_{10}(t_2) = \{c_1, c_2\}, \\
F_{11}(t_1) = \{c_1\}, & F_{11}(t_2) = \phi, \\
F_{12}(t_1) = \{c_1\}, & F_{12}(t_2) = \{c_1\}, \\
F_{13}(t_1) = \phi, & F_{13}(t_2) = \{c_2\}, \\
F_{14}(t_1) = \phi, & F_{14}(t_2) = \{c_1, c_2\}, \\
F_{15}(t_1) = \phi, & F_{15}(t_2) = \phi, \\
F_{16}(t_1) = \phi, & F_{16}(t_2) = \{c_1\},
\end{array}
$$

Are all soft sets on universal set X under the parameter set E.

$\tau = \{ (F, E)_2, (F, E)_9, (F, E)_{10}, (F, E)_{11}, (F, E)_{15} \}$ is a soft topology over X.

Soft pre-open sets are $\{ (F, E)_1, (F, E)_2, (F, E)_3, (F, E)_4, (F, E)_7, (F, E)_9, (F, E)_{10}, (F, E)_{11}, (F, E)_{12}, (F, E)_{15} \}$

Soft pre*-open sets are { $(F,E)_1$, $(F,E)_2$, $(F,E)_3$, $(F,E)_4$, $(F,E)_5$, $(F,E)_6$, $(F,E)_7$, $(F,E)_8$, $(F,E)_9$, $(F,E)_{10}$, $(F,E)_{11}$, $(F,E)_{12}$, $(F,E)_{13}$, $(F,E)_{14}$, $(F,E)_{15}$, $(F,E)_{16}$ }

soft IP*-open sets are { $(F,E)_1$, $(F,E)_2$, $(F,E)_3$, $(F,E)_4$, $(F,E)_7$, $(F,E)_9$, $(F,E)_{10}$, $(F,E)_{11}$, $(F,E)_{12}$, $(F,E)_{15}$ }.

Theorem 3.1. *If (M,E) is a soft IP*-open set such that $(N,E) \tilde{\subseteq} (M,E) \tilde{\subseteq} \tilde{s}\mathrm{cl}(N,E)$ soft IP*-open set.*

Proof. Given that $(N,E) \tilde{\subseteq} (M,E) \tilde{\subseteq} \tilde{s}\mathrm{cl}(N,E)$ and (M,E) is soft IP*-open set. Since (M,E) is a soft IP*-open set, $(M,E) \tilde{\subseteq} \{\tilde{s}\mathrm{int}(\tilde{s}\mathrm{cl}(M,E))\}\tilde{\bigcap}\{\tilde{s}\mathrm{int}^*(\tilde{s}\mathrm{cl}(M,E))\}$.

Now $\tilde{s}\mathrm{cl}(M,E) \tilde{\subseteq} \tilde{s}\mathrm{cl}(N,E) \Rightarrow \tilde{s}\mathrm{int}(\tilde{s}\mathrm{cl}(M,E)) \tilde{\subseteq} \tilde{s}\mathrm{int}(\tilde{s}\mathrm{cl}(N,E))$ and

$$\begin{aligned}(N,E) &\tilde{\subseteq} (M,E) \tilde{\subseteq} \tilde{s}\mathrm{int}(\tilde{s}\mathrm{cl}(M,E)) \tilde{\subseteq} \tilde{s}\mathrm{int}(\tilde{s}\mathrm{cl}(N,E)) \\ &\Rightarrow (N,E) \tilde{\subseteq} \tilde{s}\mathrm{int}(\tilde{s}\mathrm{cl}(N,E)) \qquad (1)\end{aligned}$$

Or

$$\tilde{s}\mathrm{cl}(M,E) \tilde{\subseteq} \tilde{s}\mathrm{cl}(N,E) \Rightarrow \tilde{s}\mathrm{int}^*(\tilde{s}\mathrm{cl}(M,E)) \tilde{\subseteq} \tilde{s}\mathrm{int}^*(\tilde{s}\mathrm{cl}(N,E))$$

and

$$\begin{aligned}(N,E) &\tilde{\subseteq} (M,E) \tilde{\subseteq} \tilde{s}\mathrm{int}^*(\tilde{s}\mathrm{cl}(M,E)) \tilde{\subseteq} \tilde{s}\mathrm{int}^*(\tilde{s}\mathrm{cl}(N,E)) \\ &\Rightarrow (N,E) \tilde{\subseteq} \tilde{s}\mathrm{int}^*(\tilde{s}\mathrm{cl}(N,E)) \qquad (2)\end{aligned}$$

By equation (1) and (2)

$$(N,E) \tilde{\subseteq} \tilde{s}\mathrm{int}(\tilde{s}\mathrm{cl}(N,E)) \text{ and } (N,E) \tilde{\subseteq} \tilde{s}\mathrm{int}^*(\tilde{s}\mathrm{cl}(N,E))$$

Therefore $(N,E) \tilde{\subseteq} \{\tilde{s}\mathrm{int}(\tilde{s}\mathrm{cl}(N,E))\}\tilde{\bigcap}\{\tilde{s}\mathrm{int}^*(\tilde{s}\mathrm{cl}(N,E))\}$

Hence (N,E) is a soft IP*-open set. □

We shall denote the family of all soft IP*-open sets (resp. soft IP*-closed sets) of a soft topological spaces (X,E,τ) by $\widetilde{IP}^*\mathrm{OSS}(X)_E$ (resp. $\widetilde{IP}^*\mathrm{CSS}(X)_E$).

Definition 3.2. Let (X,E,τ) be a soft topological space and (M,E) be a soft set over X.

(i) The soft IP*-closure of (M,E) is a soft set, $\widetilde{IP}^*\text{-cl}\,(M,E) = \tilde{\bigcap}\{(S,E) : (M,E) \tilde{\subseteq} (S,E) \text{ and } (S,E) \in \widetilde{IP}^*\mathrm{CSS}(X)_E\}$

(ii) The soft IP*-interior of (M,E) is a soft set, $\widetilde{IP}^*\text{-int}\,(M,E) = \tilde{\bigcup}\{(S,E) : (S,E) \tilde{\subseteq} (M,E) \text{ and } (S,E) \in \widetilde{IP}^*\mathrm{OSS}(X)_E\}$

Note that $\widetilde{IP}^*\text{-cl}(M,E)$ is the smallest soft IP*-closed set containing (M,E) and $\widetilde{IP}^*\text{-int}(M,E)$ is the largest soft IP*-open set cantained in (M,E).

Definition 3.3. Let (X,E,τ) is a soft topological space and (B,E) is a soft set. Then soft IP*-boundary of a soft set (B,E) is defined as

$$\widetilde{IP}^*\text{-b}(B,E) = \widetilde{IP}^*\text{-cl}(B,E)\tilde{\bigcap}\widetilde{IP}^*\text{-cl}(B,E)^c.$$

Theorem 3.2. *Every soft open set is a soft IP*-open set.*

Proof. Let (M, E) be a soft open set. By Theorem (2.1, 2.2.) every soft open set is a soft pre-open set (soft pre*-open set)

Then $(M, E) \widetilde{\subseteq} \widetilde{s}\,\mathrm{int}(\widetilde{s}\,\mathrm{cl}(M, E))$ and $(M, E) \widetilde{\subseteq} \widetilde{s}\,\mathrm{int}^*(\widetilde{s}\,\mathrm{cl}(M, E))$

$$\Rightarrow (M, E) \widetilde{\subseteq} \{\widetilde{s}\,\mathrm{int}(\widetilde{s}\,\mathrm{cl}(M, E))\}\widetilde{\bigcap}\{\widetilde{s}\,\mathrm{int}^*(\widetilde{s}\,\mathrm{cl}(M, E))\}$$

Hence (M, E) is a soft IP*-open set. □

Theorem 3.3. *Arbitrary union of soft IP*-open sets is a soft IP*-open set.*

Proof. Let $\{(M, E)_\alpha / \alpha \in i\}$ be a collection of soft IP*-open sets of a soft topological spaces (X, E, τ). Then for each α, $(M, E)_\alpha \widetilde{\subseteq} \{\widetilde{s}\,\mathrm{int}(\widetilde{s}\,\mathrm{cl}(M, E)_\alpha)\}\widetilde{\bigcap}\{\widetilde{s}\,\mathrm{int}^*(\widetilde{s}\,\mathrm{cl}(M, E)_\alpha)\}$

Since $(M, E)_\alpha \widetilde{\subseteq} \{\widetilde{s}\,\mathrm{int}(\widetilde{s}\,\mathrm{cl}(M, E)_\alpha)\}\widetilde{\bigcap}\{\widetilde{s}\,\mathrm{int}^*(\widetilde{s}\,\mathrm{cl}(M, E)_\alpha)\}$

Then

$$(M, E)_\alpha \widetilde{\subseteq} \widetilde{s}\,\mathrm{int}(\widetilde{s}\,\mathrm{cl}(M, E)_\alpha) \tag{1}$$

$$(M, E)_\alpha \widetilde{\subseteq} \widetilde{s}\,\mathrm{int}^*(\widetilde{s}\,\mathrm{cl}(M, E)_\alpha) \tag{2}$$

$$(M, E)_\alpha \widetilde{\subseteq} \{\widetilde{s}\,\mathrm{int}(\widetilde{s}\,\mathrm{cl}(M, E)_\alpha)\}\widetilde{\bigcap}\{\widetilde{s}\,\mathrm{int}^*(\widetilde{s}\,\mathrm{cl}(M, E)_\alpha)\}$$
$$\widetilde{\bigcup}(M, E)_\alpha \widetilde{\subseteq} \widetilde{\bigcup}\{\{\widetilde{s}\,\mathrm{int}(\widetilde{s}\,\mathrm{cl}(M, E)_\alpha)\}\widetilde{\bigcap}\{\widetilde{s}\,\mathrm{int}^*(\widetilde{s}\,\mathrm{cl}(M, E)_\alpha)\}\}$$
$$\widetilde{\bigcup}(M, E)_\alpha \widetilde{\subseteq} \{\widetilde{\bigcup}\widetilde{s}\,\mathrm{int}(\widetilde{s}\,\mathrm{cl}(M, E)_\alpha)\}\widetilde{\bigcap}\{\widetilde{\bigcup}\widetilde{s}\,\mathrm{int}^*(\widetilde{s}\,\mathrm{cl}(M, E)_\alpha)\}$$
$$\widetilde{\bigcup}(M, E)_\alpha \widetilde{\subseteq} \{\widetilde{s}\,\mathrm{int}(\widetilde{\bigcup}\widetilde{s}\,\mathrm{cl}(M, E)_\alpha)\}\widetilde{\bigcap}\{\widetilde{s}\,\mathrm{int}^*(\widetilde{\bigcup}\widetilde{s}\,\mathrm{cl}(M, E)_\alpha)\}$$
$$\widetilde{\bigcup}(M, E)_\alpha \widetilde{\subseteq} \{\widetilde{s}\,\mathrm{int}(\widetilde{s}\,\mathrm{cl}(\widetilde{\bigcup}(M, E)_\alpha))\}\widetilde{\bigcap}\{\widetilde{s}\,\mathrm{int}^*(\widetilde{s}\,\mathrm{cl}(\widetilde{\bigcup}(M, E)_\alpha))\}$$
$$\widetilde{\bigcup}(M, E)_\alpha \widetilde{\subseteq} \{\widetilde{s}\,\mathrm{int}(\widetilde{s}\,\mathrm{cl}(\widetilde{\bigcup}(M, E)_\alpha))\}\widetilde{\bigcap}\{\widetilde{s}\,\mathrm{int}^*(\widetilde{s}\,\mathrm{cl}(\widetilde{\bigcup}(M, E)_\alpha))\}$$

And hence Arbitrary union of soft IP*-open sets is a soft IP*-open set. □

Theorem 3.4. *Let (X, E, τ) be a soft topological space and (M, E) be a soft set over X. Then,*

(i) $(M, E) \in \widetilde{IP}^*\mathrm{CSS}(X)_E$ *if and only if* $(M, E) = \widetilde{IP}^*\text{-}cl(M, E)$
(ii) $(M, E) \in \widetilde{IP}^*\mathrm{OSS}(X)_E$ *if and only if* $(M, E) = \widetilde{IP}^*\text{-}int(M, E)$
(iii) $\widetilde{IP}^*\text{-}cl(\varnothing, E) = (\varnothing, E)$ *and* $\widetilde{IP}^*\text{-}cl(X, E) = (X, E)$
(iv) $\widetilde{IP}^*\text{-}int(\varnothing, E) = (\varnothing, E)$ *and* $\widetilde{IP}^*\text{-}int(X, E) = (X, E)$
(v) $\widetilde{IP}^*\text{-}cl(\widetilde{IP}^*\text{-}cl(M, E)) = \widetilde{IP}^*\text{-}cl(M, E)$
(vi) $\widetilde{IP}^*\text{-}int(\widetilde{IP}^*\text{-}int(M, E)) = \widetilde{IP}^*\text{-}int(M, E)$

Proof. Let (M, E) be a soft set over X.

(i) Let (M, E) be a soft IP*-closed set. Then it is the smallest soft IP*-closed set containing it self, and hence $(M, E) = \widetilde{IP}^*\text{-cl}(M, E)$. Conversely, let $(M, E) = \widetilde{IP}^*\text{-cl}(M, E)$, $\widetilde{IP}^*\text{-cl}(M, E)$ being the intersection of soft IP*-closed set is soft IP*-closed so $\widetilde{IP}^*\text{-cl}(M, E) \in \widetilde{IP}^*\mathrm{CSS}(X)_E \Longrightarrow (M, E) \in \widetilde{IP}^*\mathrm{CSS}(X)_E$.

(ii) Let (M, E) be a soft IP*-open set. Then it is the smallest soft IP*-open set contained in (M, E) and hence $(M, E) = \widetilde{IP}^*$-int$(M, E)$. Conversely, let $(M, E) = \widetilde{IP}^*$-int$(M, E)$, $\widetilde{IP}^*$-int(M, E) being the union of soft IP*-open set is soft IP*-open so $\widetilde{IP}^*$-int$(M, E) \in \widetilde{IP}^*\mathrm{OSS}(X)_E \Longrightarrow (M, E) \in \widetilde{IP}^*\mathrm{OSS}(X)_E$.

(iii) Since $(\varnothing, E)$ and (X, E) are soft IP*-closed sets, so by (i) $\widetilde{IP}^*$-cl$(\varnothing, E) = (\varnothing, E)$ and $\widetilde{IP}^*$-cl$(X, E) = (X, E)$

(iv) Since $(\varnothing, E)$ and (X, E) are soft IP*-open set, so by (ii) $\widetilde{IP}^*$-int$(\varnothing, E) = (\varnothing, E)$ and $\widetilde{IP}^*$-int$(X, E) = (X, E)$

(v) Since $\widetilde{IP}^*$-cl$(M, E) \in \widetilde{IP}^*\mathrm{CSS}(X)_E$ by (i) $(M, E) \in \widetilde{IP}^*\mathrm{CSS}(X)_E$ iff $(M, E) = \widetilde{IP}^*$-cl$(M, E)$ therefore $\widetilde{IP}^*$-cl$(\widetilde{IP}^*$-cl$(M, E)) = \widetilde{IP}^*$-cl$(M, E)$

(vi) Since $\widetilde{IP}^*$-int$(M, E) \in \widetilde{IP}^*\mathrm{OSS}(X)_E$ by (ii) $(M, E) \in \widetilde{IP}^*\mathrm{OSS}(X)_E$ iff $(M, E) = \widetilde{IP}^*$-int$(M, E)$ therefore $\widetilde{IP}^*$-int$(\widetilde{IP}^*$-int$(M, E)) = \widetilde{IP}^*$-int$(M, E)$. □

Theorem 3.5. *If (B, E) is a soft subset of a soft topological spaces (X, E, τ) then the following statements hold*

(i) $\widetilde{IP}^*$-$b(B, E) = \widetilde{IP}^*$-$b(X \setminus (B, E))$

(ii) $\widetilde{IP}^*$-$b(B, E) = \widetilde{IP}^*$-$cl(B, E) \setminus \widetilde{IP}^*$-$int(B, E)$

(iii) $\widetilde{IP}^*$-$b(B, E) \widetilde{\bigcap} \widetilde{IP}^*$-$int(B, E) = \phi$

(iv) $\widetilde{IP}^*$-$b(B, E) \widetilde{\bigcup} \widetilde{IP}^*$-$int(B, E) = \widetilde{IP}^*$-$cl(B, E)$.

Definition 3.4. If (B, E) is a soft subset of a soft topological spaces (X, E, τ). Then a point $x \in X$ is called soft IP*-limit point of a soft set $(B, E) \widetilde{\subseteq} X$ if every soft IP*-open set $(G, E) \widetilde{\subseteq} X$ containing x contains a point other than x.

Definition 3.5. The soft set of all soft IP*-limit point of (B, E) is called soft IP*-derived set of (B, E) and is denoted by $\widetilde{IP}^*$-d(B, E).

Definition 3.6. A soft subset (B, E) of a soft topological space (X, E, τ) is said to be a soft IP*-neighborhood (Briefly $\widetilde{IP}^*$-nbd) of a point $p \in X$ if there exists an soft IP*-open set (M, E) such that $p \in (M, E) \widetilde{\subseteq} (B, E)$. The class of soft IP*-neighborhood of $p \in X$ is called the soft IP*-neighborhood system of p and denoted by $\widetilde{IP}^*$-N_p.

Theorem 3.6. *If (M, E) and (N, E) are two soft subset of a soft topological space (X, E, τ). Then, (i) $(M, E) \widetilde{\subseteq} (N, E)$ then $\widetilde{IP}^*$-$d(M, E) \widetilde{\subseteq} \widetilde{IP}^*$-$d(N, E)$.*

(ii) (M, E) is an soft IP-closed set if and only if it contains each of its soft IP*-limit points.*

(iii) $\widetilde{IP}^*$-$cl(M, E) = (M, E) \widetilde{\bigcup} \widetilde{IP}^*$-$d(M, E)$.

Proof. (i). Let $(M, E) \widetilde{\subseteq} (N, E)$ and

$P \in \widetilde{IP}^*$-d(M, E) iff $((G, E) - \{p\}) \widetilde{\bigcap} (M, E) \neq \varnothing$ for every soft IP*-open set (G, E) containing p. But $(N, E) \widetilde{\supseteq} (M, E)$,

Hence $((G, E) - \{p\}) \widetilde{\bigcap} (N, E) \widetilde{\supseteq} ((G, E) - \{p\}) \widetilde{\bigcap} (M, E) \neq \varnothing$

So $p \in \widetilde{IP}^*$-d$(M, E) \Longrightarrow p \in \widetilde{IP}^*$-d$(N, E)$, i.e. $\widetilde{IP}^*$-d$(M, E) \widetilde{\subseteq} \widetilde{IP}^*$-d$(N, E)$.

(ii) Suppose (M,E) is soft IP*-closed and, let $p \notin (M,E)$, i.e. $p \in (M,E)^c$. But $(M,E)^c$, the soft complement of a soft IP*-closed, is soft IP*-open;

Hence $p \notin \widetilde{IP}^*$-d$(M,E)$ for $(M,E)^c$ is an soft IP*-open set such that

$p \in (M,E)^c$ and $(M,E)^c \widetilde{\bigcap}(M,E) = \varnothing$. Thus $\widetilde{IP}^*$-d$(M,E) \widetilde{\subseteq} (M,E)$ if (M,E) is closed.

Now assume $\widetilde{IP}^*$-d$(M,E) \widetilde{\subseteq} (M,E)$; we show that $(M,E)^c$ is soft IP*-open set. Let $p \in (M,E)^c$, then $p \notin \widetilde{IP}^*$-d$(M,E)$, so $\exists$ a soft IP*-open set (G,E)

Such that $p \in (G,E)$ and $((G,E) - \{p\})\widetilde{\bigcap}(M,E) = \varnothing$

But $p \notin (M,E)$; hence

$$(G,E)\widetilde{\bigcap}(M,E) = ((G,E) - \{p\})\widetilde{\bigcap}(M,E) = \varnothing$$

so $(G,E) \widetilde{\subseteq} (M,E)^c$. Thus p is an soft IP*-interior point of $(M,E)^c$, and so $(M,E)^c$ is soft IP*-open.

(iii) Since $(M,E) \widetilde{\subseteq} \widetilde{IP}^*$-cl$(M,E)$ and $\widetilde{IP}^*$-cl(M,E) is soft IP*-closed,

$$\widetilde{IP}^*\text{-d}(M,E) \widetilde{\subseteq} \widetilde{IP}^*\text{-d}[\widetilde{IP}^*\text{-cl}(M,E)] \widetilde{\subseteq} \widetilde{IP}^*\text{-cl}(M,E)$$

And hence $(M,E)\widetilde{\bigcup}\widetilde{IP}^*$-d$(M,E) \widetilde{\subseteq} \widetilde{IP}^*$-cl$(M,E)$.

But $(M,E)\widetilde{\bigcup}\widetilde{IP}^*$-d$(M,E)$ is a soft IP*-closed set containing (M,E), so $(M,E) \widetilde{\subseteq} \widetilde{IP}^*$-cl$(M,E) \widetilde{\subseteq} (M,E)\widetilde{\bigcup}\widetilde{IP}^*$-d$(M,E)$. Thus $\widetilde{IP}^*$-cl$(M,E) = (M,E)\widetilde{\bigcup}(\widetilde{IP}^*$-d$(M,E))$. □

Theorem 3.7. *Intersection of any two soft IP*-neighborhoods of a point p is also a soft IP*-neighborhood of a point p.*

Proof. Let (N,E) and (M,E) be a soft IP*-neighborhoods of p, so there exists soft IP*-open sets (G,E) and (H,E) such that $p \in (G,E) \widetilde{\subseteq} (N,E)$

And

$p \in (H,E) \widetilde{\subseteq} (M,E)$

Hence $p \in (G,E)\widetilde{\bigcap}(H,E) \widetilde{\subseteq} (N,E)\widetilde{\bigcap}(M,E)$, and

$(G,E)\widetilde{\bigcap}(H,E)$ is soft IP*-open, or $(N,E)\widetilde{\bigcap}(M,E)$ is a soft IP*-neighborhood of p. □

Theorem 3.8. *Any soft superset (M,E) of a soft IP*-neighborhood (N,E) of a point p is also a soft IP*-neighborhood of p.*

Proof. Let (N,E) is a soft IP*-neighborhood of p, so there exists a soft -open set (G,E) such that $p \in (G,E) \widetilde{\subseteq} (N,E)$. By hypothesis, $(N,E) \widetilde{\subseteq} (M,E)$, so $p \in (G,E) \widetilde{\subseteq} (N,E) \widetilde{\subseteq} (M,E)$ which implies $p \in (G,E) \widetilde{\subseteq} (M,E)$ and hence (M,E) is a soft IP*-neighborhood of p. □

Proposition 3.1. *Let (X,E,τ) be a soft discrete topological space. Then soft IP*-open set of every soft singleton set of X is empty*

Solution. Let (M, E) be an soft subset of soft discrete topological spaces and p be any point in X. Every soft subset of a soft discrete space is soft IP*-open. Hence the soft singleton set $(N, E) = \{p\}$ is an soft IP*-open subset of X.

But $p \in (N, E)$ and $(N, E)\widetilde{\bigcap}(M, E) \underline{=} (\{p\}\widetilde{\bigcap}(M, E)) \widetilde{\subseteq} \{p\}$

Hence, by the above problem, $p \notin \widetilde{IP}^{*}$-d$(M, E)$ for every $p \in X$

i.e. $\widetilde{IP}^{*}$-d$(M, E) = \emptyset$. □

Proposition 3.2. *Let (M, E) be a soft subset of soft topological space (X, E, τ). When will a point $p \in X$ not be a soft IP*-limit point of (M, E).*

Solution. Let p is not a soft IP*-limit point of (M, E) if there exists a soft IP*-open set (N, E) such that $p \in (N, E)$ and

$$((N, E) - \{p\})\widetilde{\bigcap}(M, E) = \emptyset.$$

□

4. Conclusion

In this paper we introduced the concept of soft IP*-open set in soft topological space and some of their properties are studied. We also introduced soft IP*-boundary, soft IP*-limit point, soft IP*-neighborhood and their properties. In the end, we hope that, this paper is just a beginning of a new structure it will be necessary to carry out more theoretical research to promote a general framework for the practical application.

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